9.4 AREAS AND LENGTHS IN POLAR COORDINATES

A Click here for answers.

S Click here for solutions.

1–8 ▪ Find the area of the region that is bounded by the given curve and lies in the specified sector.

1. $r = \theta, \quad 0 \leqslant \theta \leqslant \pi$

2. $r = e^{\theta}, \quad -\pi/2 \leqslant \theta \leqslant \pi/2$

3. $r = 2\cos\theta, \quad 0 \leqslant \theta \leqslant \pi/6$

4. $r = 1/\theta, \quad \pi/6 \leqslant \theta \leqslant 5\pi/6$

5. $r = \sin 2\theta, \quad 0 \leqslant \theta \leqslant \pi/6$

6. $r = \cos 3\theta, \quad -\pi/12 \leqslant \theta \leqslant \pi/12$

7. $r = 3\sin\theta, \quad \pi/4 \leqslant \theta \leqslant 3\pi/4$

8. $r = \theta^2, \quad \pi/2 \leqslant \theta \leqslant 3\pi/2$

9–16 ▪ Sketch the curve and find the area that it encloses.

9. $r = 5\sin\theta$

10. $r = 4 - \sin\theta$

11. $r = \sin 3\theta$

12. $r = 4(1 - \cos\theta)$

13. $r = 2\cos\theta$

14. $r = 1 + \sin\theta$

15. $r = 3 - \cos\theta$

16. $r = \sin 4\theta$

17. Graph the curve $r = 2 + \cos 6\theta$ and find the area that it encloses.

18. The curve with polar equation $r = 2\sin\theta\cos^2\theta$ is called a **bifolium**. Graph it and find the area that it encloses.

19–22 ▪ Find the area of the region enclosed by one loop of the curve.

19. $r = \cos 3\theta$

20. $r = 3\sin 2\theta$

21. $r = \sin 5\theta$

22. $r = 2 + 3\cos\theta$ (inner loop)

23–24 ▪ Find the area of the region that lies inside the first curve and outside the second curve.

23. $r = 1 - \cos\theta, \quad r = \frac{3}{2}$

24. $r = 3\cos\theta, \quad r = 2 - \cos\theta$

25. Find the area inside the larger loop and outside the smaller loop of the limaçon $r = 3 + 4\sin\theta$.

26. Graph the hippopede $r = \sqrt{1 - 0.8\sin^2\theta}$ and the circle $r = \sin\theta$ and find the exact area of the region that lies inside both curves.

27–32 ▪ Find the length of the polar curve.

27. $r = 5\cos\theta, \quad 0 \leqslant \theta \leqslant 3\pi/4$

28. $r = 2^{\theta}, \quad 0 \leqslant \theta \leqslant 2\pi$

29. $r = 1 + \cos\theta$

30. $r = e^{-\theta}, \quad 0 \leqslant \theta \leqslant 3\pi$

31. $r = \cos^2(\theta/4)$

32. $r = \cos^2(\theta/2)$

33–34 ▪ Use a calculator or computer to find the length of the loop correct to four decimal places.

33. One loop of the four-leaved rose $r = \cos 2\theta$

34. The loop of the conchoid $r = 4 + 2\sec\theta$

9.4 ANSWERS

E Click here for exercises. S Click here for solutions.

1. $\frac{1}{6}\pi^3$

2. $\frac{1}{4}\left(e^{\pi} - e^{-\pi}\right)$

3. $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$

4. $\frac{12}{5\pi}$

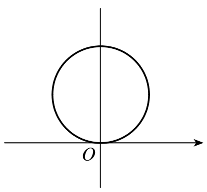
5. $\frac{4\pi - 3\sqrt{3}}{96}$

6. $\frac{1}{24}(\pi + 2)$

7. $\frac{9}{8}(\pi + 2)$

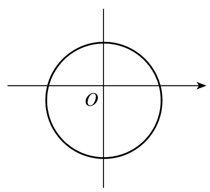
8. $\frac{121}{160}\pi^5$

9.



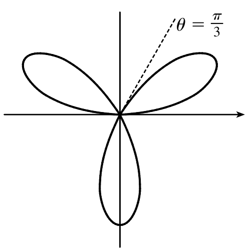
$\frac{25}{4}\pi$

10.



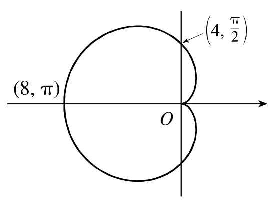
$\frac{33\pi}{2}$

11.



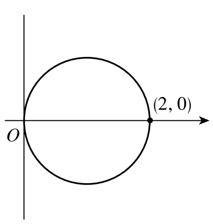
$\frac{\pi}{4}$

12.



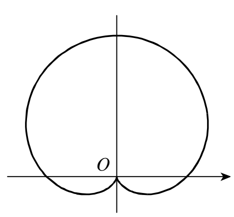
24π

13.



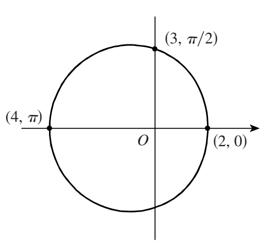
π

14.



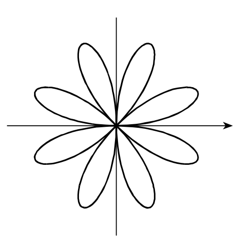
$\frac{3\pi}{2}$

15.



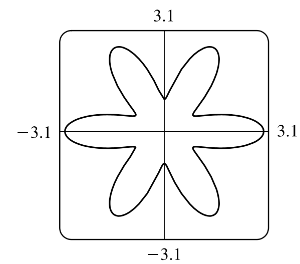
$\frac{19\pi}{2}$

16.



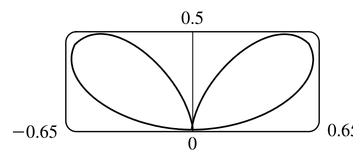
$\frac{\pi}{2}$

17.



$\frac{9\pi}{2}$

18.



$\frac{\pi}{8}$

19. $\frac{\pi}{12}$

20. $\frac{9\pi}{8}$

21. $\frac{\pi}{20}$

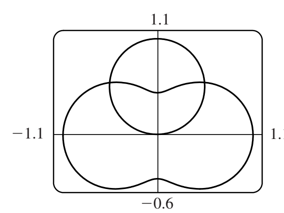
22. $\frac{17}{2}\cos^{-1}\left(\frac{2}{3}\right) - 3\sqrt{5}$

23. $\frac{9\sqrt{3}}{8} - \frac{1}{4}\pi$

24. $3\sqrt{3}$

25. $34\sin^{-1}\left(\frac{3}{4}\right) + 9\sqrt{7}$

26.



$\frac{3}{10}\pi - \frac{1}{10}\arcsin\frac{\sqrt{5}}{3} - \frac{1}{5}\sqrt{5}$

27. $\frac{15}{4}\pi$

28. $\dfrac{\sqrt{1 + \ln^2 2}\,\left(2^{2\pi} - 1\right)}{\ln 2}$

29. 8

30. $\sqrt{2}\left(1 - e^{-3\pi}\right)$

31. $\frac{16}{3}$

32. 4

33. 2.4221

34. 5.8128

9.4 SOLUTIONS

E Click here for exercises.

1. $A = \int_0^{\pi} \frac{1}{2} r^2\, d\theta = \int_0^{\pi} \frac{1}{2}\theta^2\, d\theta = \left[\frac{1}{6}\theta^3\right]_0^{\pi} = \frac{1}{6}\pi^3$

2. $A = \int_{-\pi/2}^{\pi/2} \frac{1}{2} e^{2\theta}\, d\theta = \left[\frac{1}{4}e^{2\theta}\right]_{-\pi/2}^{\pi/2} = \frac{1}{4}\left(e^{\pi} - e^{-\pi}\right)$

3. $A = \int_0^{\pi/6} \frac{1}{2}(2\cos\theta)^2\, d\theta = \int_0^{\pi/6} (1+\cos 2\theta)\, d\theta$
$= \left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/6} = \frac{\pi}{6} + \frac{\sqrt{3}}{4}$

4. $A = \int_{\pi/6}^{5\pi/6} \frac{1}{2}(1/\theta)^2\, d\theta = \left[-1/(2\theta)\right]_{\pi/6}^{5\pi/6} = \frac{12}{5\pi}$

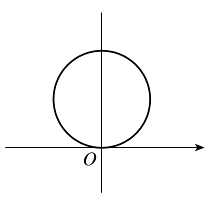
5. $A = \int_0^{\pi/6} \frac{1}{2}\sin^2 2\theta\, d\theta = \frac{1}{4}\int_0^{\pi/6}(1-\cos 4\theta)\, d\theta$
$= \left[\frac{1}{4}\theta - \frac{1}{16}\sin 4\theta\right]_0^{\pi/6} = \frac{4\pi - 3\sqrt{3}}{96}$

6. $A = 2\int_0^{\pi/12} \frac{1}{2}\cos^2 3\theta\, d\theta = \frac{1}{2}\int_0^{\pi/12}(1+\cos 6\theta)\, d\theta$
$= \frac{1}{2}\left[\theta + \frac{1}{6}\sin 6\theta\right]_0^{\pi/12} = \frac{1}{24}(\pi + 2)$

7. $A = \int_{\pi/4}^{3\pi/4} \frac{1}{2}(3\sin\theta)^2\, d\theta = 2\int_{\pi/4}^{\pi/2} \frac{9}{4}(1-\cos 2\theta)\, d\theta$
$= \frac{9}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_{\pi/4}^{\pi/2} = \frac{9}{8}(\pi + 2)$

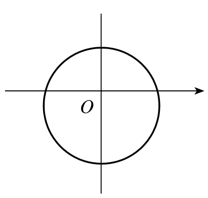
8. $A = \int_{\pi/2}^{3\pi/2} \frac{1}{2}\left(\theta^2\right)^2\, d\theta = \left[\frac{1}{10}\theta^5\right]_{\pi/2}^{3\pi/2} = \frac{121}{160}\pi^5$

9.



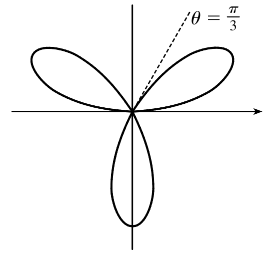
$A = \int_0^{\pi} \frac{1}{2}(5\sin\theta)^2\, d\theta = \frac{25}{4}\int_0^{\pi}(1-\cos 2\theta)\, d\theta$
$= \frac{25}{4}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi} = \frac{25}{4}\pi$

10.



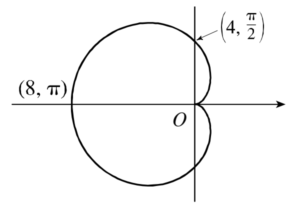
$A = 2\int_{-\pi/2}^{\pi/2} \frac{1}{2}(4-\sin\theta)^2\, d\theta$
$= \int_{-\pi/2}^{\pi/2}\left(16 - 8\sin\theta + \sin^2\theta\right) d\theta$
$= \int_{-\pi/2}^{\pi/2}\left(16 + \sin^2\theta\right) d\theta$ [by Theorem 5.5.7(b)]
$= 2\int_0^{\pi/2}\left(16 + \sin^2\theta\right) d\theta$ [by Theorem 5.5.7(a)]
$= 2\int_0^{\pi/2}\left[16 + \frac{1}{2}(1-\cos 2\theta)\right] d\theta$
$= 2\left[\frac{33}{2}\theta - \frac{1}{4}\sin 2\theta\right]_0^{\pi/2} = \frac{33\pi}{2}$

11.



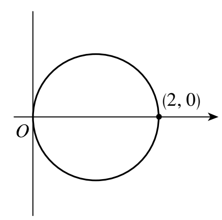
$A = 6\int_0^{\pi/6} \frac{1}{2}\sin^2 3\theta\, d\theta = 3\int_0^{\pi/6}\frac{1}{2}(1-\cos 6\theta)\, d\theta$
$= \frac{3}{2}\left[\theta - \frac{1}{6}\sin 6\theta\right]_0^{\pi/6} = \frac{\pi}{4}$

12.



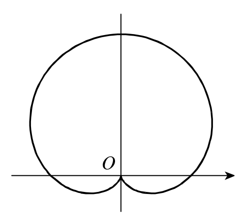
$A = 2\int_0^{\pi} \frac{1}{2}\left[4(1-\cos\theta)\right]^2\, d\theta$
$= 16\int_0^{\pi}\left(1 - 2\cos\theta + \cos^2\theta\right) d\theta$
$= 8\int_0^{\pi}(3 - 4\cos\theta + \cos 2\theta)\, d\theta$
$= 4\left[6\theta - 8\sin\theta + \sin 2\theta\right]_0^{\pi} = 24\pi$

13.



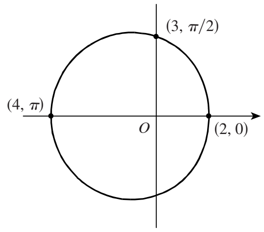
$A = 2\int_0^{\pi/2} \frac{1}{2}(2\cos\theta)^2\, d\theta = 2\int_0^{\pi/2}(1+\cos 2\theta)\, d\theta$
$= 2\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = \pi$

14.



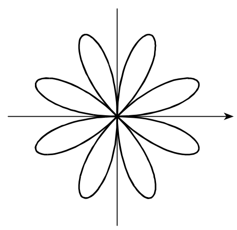
$A = 2\int_{-\pi/2}^{\pi/2} \frac{1}{2}(1+\sin\theta)^2\, d\theta$
$= \int_{-\pi/2}^{\pi/2}\left(1 + 2\sin\theta + \sin^2 2\theta\right) d\theta$
$= \left[\theta - 2\cos\theta\right]_{-\pi/2}^{\pi/2} + \int_{-\pi/2}^{\pi/2}\frac{1}{2}(1-\cos 2\theta)\, d\theta$
$= \pi + \frac{1}{2}\left[\theta - \frac{1}{2}\sin 2\theta\right]_{-\pi/2}^{\pi/2} = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

15.



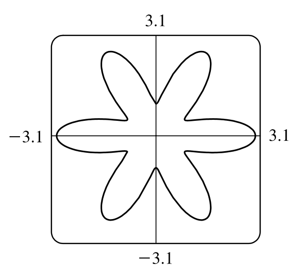
$$A = 2\int_0^{\pi} \tfrac{1}{2}(3-\cos\theta)^2\,d\theta$$
$$= \int_0^{\pi}\left(9 - 6\cos\theta + \cos^2\theta\right)d\theta$$
$$= \left[9\theta - 6\sin\theta + \tfrac{1}{2}\theta + \tfrac{1}{4}\sin 2\theta\right]_0^{\pi} = \tfrac{19\pi}{2}$$

16.



$$A = 8\int_0^{\pi/4}\tfrac{1}{2}\sin^2 4\theta\,d\theta = 2\int_0^{\pi/4}(1-\cos 8\theta)\,d\theta$$
$$= \left[2\theta - \tfrac{1}{4}\sin 8\theta\right]_0^{\pi/4} = \tfrac{\pi}{2}$$

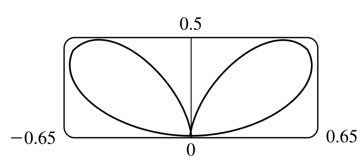
17.



By symmetry, the total area is twice the area enclosed above the polar axis, so

$$A = 2\int_0^{\pi}\tfrac{1}{2}r^2\,d\theta = \int_0^{\pi}[2+\cos 6\theta]^2\,d\theta$$
$$= \int_0^{\pi}\left(4 + 4\cos 6\theta + \cos^2 6\theta\right)d\theta$$
$$= \left[4\theta + 4\left(\tfrac{1}{6}\sin 6\theta\right) + \left(\tfrac{1}{24}\sin 12\theta + \tfrac{1}{2}\theta\right)\right]_0^{\pi}$$
$$= 4\pi + \tfrac{\pi}{2} = \tfrac{9\pi}{2}$$

18.



Note that the entire curve $r = 2\sin\theta\cos^2\theta$ is generated by $\theta \in [0,\pi]$. The radius is positive on this interval, so the area enclosed is

$$A = \int_0^{\pi}\tfrac{1}{2}r^2\,d\theta = \int_0^{\pi}\tfrac{1}{2}\left(2\sin\theta\cos^2\theta\right)^2 d\theta$$
$$= 2\int_0^{\pi}\sin^2\theta\cos^4\theta\,d\theta = 2\int_0^{\pi}(\sin\theta\cos\theta)^2\cos^2\theta\,d\theta$$
$$= 2\int_0^{\pi}\left(\tfrac{1}{2}\sin 2\theta\right)^2\cos^2\theta\,d\theta$$
$$= \tfrac{1}{4}\int_0^{\pi}\sin^2 2\theta\,(\cos 2\theta + 1)\,d\theta$$
$$= \tfrac{1}{4}\left[\int_0^{\pi}\sin^2 2\theta\cos 2\theta\,d\theta + \int_0^{\pi}\sin^2 2\theta\,d\theta\right]$$
$$= \tfrac{1}{4}\left[\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 4\theta\right]_0^{\pi} \text{ (the first integral vanishes) } = \tfrac{\pi}{8}$$

19. $A = 2\int_0^{\pi/6}\tfrac{1}{2}\cos^2 3\theta\,d\theta = \tfrac{1}{2}\int_0^{\pi/6}(1+\cos 6\theta)\,d\theta$

$$= \tfrac{1}{2}\left[\theta + \tfrac{1}{6}\sin 6\theta\right]_0^{\pi/6} = \tfrac{\pi}{12}$$

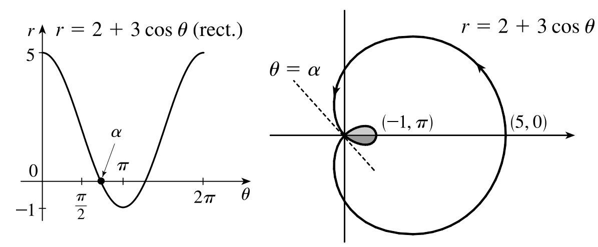
20. $A = 2\int_0^{\pi/4}\tfrac{1}{2}(3\sin 2\theta)^2\,d\theta = \tfrac{9}{2}\int_0^{\pi/4}(1-\cos 4\theta)\,d\theta$

$$= \tfrac{9}{2}\left[\theta - \tfrac{1}{4}\sin 4\theta\right]_0^{\pi/4} = \tfrac{9\pi}{8}$$

21. $A = \int_0^{\pi/5}\tfrac{1}{2}\sin^2 5\theta\,d\theta = \tfrac{1}{4}\int_0^{\pi/5}(1-\cos 10\theta)\,d\theta$

$$= \tfrac{1}{4}\left[\theta - \tfrac{1}{10}\sin 10\theta\right]_0^{\pi/5} = \tfrac{\pi}{20}$$

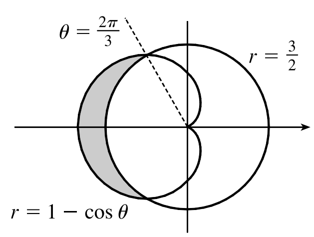
22.



$2 + 3\cos\theta = 0 \;\Rightarrow\; \cos\theta = -\tfrac{2}{3} \;\Rightarrow\; \theta = \cos^{-1}\left(-\tfrac{2}{3}\right)$ or $2\pi - \cos^{-1}\left(-\tfrac{2}{3}\right)$. Let $\alpha = \cos^{-1}\left(-\tfrac{2}{3}\right)$. Then

$$A = 2\int_{\alpha}^{\pi}\tfrac{1}{2}(2+3\cos\theta)^2\,d\theta$$
$$= \int_{\alpha}^{\pi}\left(4 + 12\cos\theta + 9\cos^2\theta\right)d\theta$$
$$= \int_{\alpha}^{\pi}\left(\tfrac{17}{2} + 12\cos\theta + \tfrac{9}{2}\cos 2\theta\right)d\theta$$
$$= \left[\tfrac{17}{2}\theta + 12\sin\theta + \tfrac{9}{4}\sin 2\theta\right]_{\alpha}^{\pi}$$
$$= \tfrac{17}{2}(\pi - \alpha) - 12\sin\alpha - \tfrac{9}{2}\sin\alpha\cos\alpha$$
$$= \tfrac{17}{2}\left[\pi - \cos^{-1}\left(-\tfrac{2}{3}\right)\right] - 12\left(\tfrac{\sqrt{5}}{3}\right) - \tfrac{9}{2}\left(\tfrac{\sqrt{5}}{3}\right)\left(-\tfrac{2}{3}\right)$$
$$= \tfrac{17}{2}\cos^{-1}\left(\tfrac{2}{3}\right) - 3\sqrt{5}$$

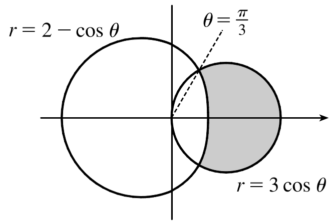
23.



$1 - \cos\theta = \tfrac{3}{2} \;\Rightarrow\; \cos\theta = -\tfrac{1}{2} \;\Rightarrow\; \theta = \tfrac{2\pi}{3}$ or $\tfrac{4\pi}{3} \;\Rightarrow$

$$A = 2\int_{2\pi/3}^{\pi}\tfrac{1}{2}\left[(1-\cos\theta)^2 - \left(\tfrac{3}{2}\right)^2\right]d\theta$$
$$= \int_{2\pi/3}^{\pi}\left(-\tfrac{5}{4} - 2\cos\theta + \cos^2\theta\right)d\theta$$
$$= \left[-\tfrac{5}{4}\theta - 2\sin\theta\right]_{2\pi/3}^{\pi} + \tfrac{1}{2}\int_{2\pi/3}^{\pi}(1+\cos 2\theta)\,d\theta$$
$$= -\tfrac{5}{12}\pi + \sqrt{3} + \tfrac{1}{2}\left[\theta + \tfrac{1}{2}\sin 2\theta\right]_{2\pi/3}^{\pi}$$
$$= -\tfrac{5}{12}\pi + \sqrt{3} + \tfrac{1}{6}\pi + \tfrac{\sqrt{3}}{8} = \tfrac{9\sqrt{3}}{8} - \tfrac{1}{4}\pi$$

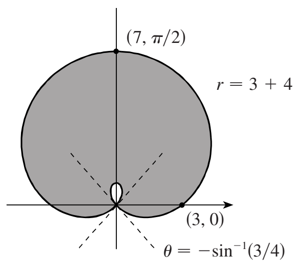
24.



$3\cos\theta = 2 - \cos\theta \quad\Rightarrow\quad \cos\theta = \frac{1}{2} \quad\Rightarrow\quad \theta = \pm\frac{\pi}{3} \quad\Rightarrow$

$$\begin{aligned} A &= 2\int_0^{\pi/3} \tfrac{1}{2}\left[(3\cos\theta)^2 - (2-\cos\theta)^2\right] d\theta \\ &= \int_0^{\pi/3} \left(8\cos^2\theta + 4\cos\theta - 4\right) d\theta \\ &= \int_0^{\pi/3} (4\cos 2\theta + 4\cos\theta)\, d\theta \\ &= \left[2\sin 2\theta + 4\sin\theta\right]_0^{\pi/3} = 3\sqrt{3} \end{aligned}$$

25.



The curve crosses itself when $3 + 4\sin\theta = 0 \quad\Leftrightarrow$ $\sin\theta = -\frac{3}{4}$. Letting $\alpha = \sin^{-1}\frac{3}{4}$, the desired area is

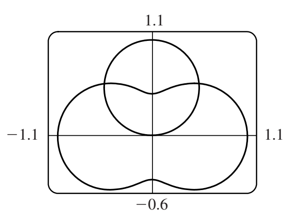
$$A = 2\left[\int_{-\alpha}^{\pi/2} \tfrac{1}{2}(3+4\sin\theta)^2\, d\theta - \int_{-\pi/2}^{-\alpha} \tfrac{1}{2}(3+4\sin\theta)^2\right] d\theta$$

Now

$\int (3+4\sin\theta)^2\, d\theta = 9\theta - 24\cos\theta + 8\theta - 4\sin 2\theta + C$, so

$A = 34\alpha + 48\cos\alpha - 16\sin\alpha\cos\alpha = 34\sin^{-1}\left(\frac{3}{4}\right) + 9\sqrt{7}$.

26.



The points of intersection occur where

$\sqrt{1 - 0.8\sin^2\theta} = \sin\theta \quad\Leftrightarrow\quad 1.8\sin^2\theta = 1 \quad\Leftrightarrow$

$\theta = \arcsin\sqrt{\frac{5}{9}}$ $\left(= \alpha, \text{ so } \cos\alpha = \frac{2}{3}\right)$. So the area is

$$\begin{aligned} A &= 2\int_0^{\alpha} \tfrac{1}{2}\sin^2\theta d\theta + 2\int_{\alpha}^{\pi/2} \tfrac{1}{2}\left(\sqrt{1-0.8\sin^2\theta}\right)^2 d\theta \\ &= \left[\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right]_0^{\alpha} + \left[\theta - 0.8\left(\tfrac{1}{2}\theta - \tfrac{1}{4}\sin 2\theta\right)\right]_{\alpha}^{\pi/2} \\ &= \tfrac{1}{2}\alpha - \tfrac{1}{4}(2\sin\alpha\cos\alpha) + 0.6\cdot\tfrac{\pi}{2} \\ &\qquad - \left[0.6\alpha + 0.2(2\sin\alpha\cos\alpha)\right] \\ &= \tfrac{1}{2}\arcsin\tfrac{\sqrt{5}}{3} - \tfrac{1}{2}\tfrac{\sqrt{5}}{3}\tfrac{2}{3} + 0.3\pi \\ &\qquad -0.6\arcsin\tfrac{\sqrt{5}}{3} - 0.4\cdot\tfrac{\sqrt{5}}{3}\tfrac{2}{3} \\ &= \tfrac{3}{10}\pi - \tfrac{1}{10}\arcsin\tfrac{\sqrt{5}}{3} - \tfrac{1}{5}\sqrt{5} \approx 0.411 \end{aligned}$$

27. $L = \int_a^b \sqrt{r^2 + (dr/d\theta)^2}\, d\theta$

$= \int_0^{3\pi/4} \sqrt{(5\cos\theta)^2 + (-5\sin\theta)^2}\, d\theta$

$= 5\int_0^{3\pi/4} \sqrt{\cos^2\theta + \sin^2\theta}\, d\theta$

$= 5\int_0^{3\pi/4} d\theta = \frac{15}{4}\pi$

28. $L = \int_a^b \sqrt{r^2 + (dr/d\theta)^2}\, d\theta$

$= \int_0^{2\pi} \sqrt{(2^\theta)^2 + [(\ln 2)\, 2^\theta]^2}\, d\theta = \int_0^{2\pi} 2^\theta\sqrt{1 + \ln^2 2}\, d\theta$

$= \left[\sqrt{1+\ln^2 2}\left(\dfrac{2^\theta}{\ln 2}\right)\right]_0^{2\pi} = \dfrac{\sqrt{1+\ln^2 2}\,(2^{2\pi} - 1)}{\ln 2}$

29. $L = 2\int_0^{\pi} \sqrt{(1+\cos\theta)^2 + (-\sin\theta)^2}\, d\theta$

$= 2\sqrt{2}\int_0^{\pi} \sqrt{1+\cos\theta}\, d\theta = 2\sqrt{2}\int_0^{\pi} \sqrt{2\cos^2(\theta/2)}\, d\theta$

$= [8\sin(\theta/2)]_0^{\pi} = 8$

30. $L = \int_0^{3\pi} \sqrt{(e^{-\theta})^2 + (-e^{-\theta})^2}\, d\theta$

$= \sqrt{2}\int_0^{3\pi} e^{-\theta}\, d\theta = \sqrt{2}\left(1 - e^{-3\pi}\right)$

31. $L = 2\int_0^{2\pi} \sqrt{\cos^8\left(\frac{1}{4}\theta\right) + \cos^6\left(\frac{1}{4}\theta\right)\sin^2\left(\frac{1}{4}\theta\right)}\, d\theta$

$= 2\int_0^{2\pi} \left|\cos^3\left(\frac{1}{4}\theta\right)\right| \sqrt{\cos^2\left(\frac{1}{4}\theta\right) + \sin^2\left(\frac{1}{4}\theta\right)}\, d\theta$

$= 2\int_0^{2\pi} \left|\cos^3\left(\frac{1}{4}\theta\right)\right| d\theta$

$= 8\int_0^{\pi/2} \cos^3 u\, du$ (where $u = \frac{1}{4}\theta$)

$= 8\left[\sin u - \frac{1}{3}\sin^3 u\right]_0^{\pi/2} = \frac{16}{3}$

Note that the curve is retraced after every interval of length 4π.

32. $L = 2\int_0^{\pi} \sqrt{\left[\cos^2\left(\frac{1}{2}\theta\right)\right]^2 + \left[-\cos\left(\frac{1}{2}\theta\right)\sin\left(\frac{1}{2}\theta\right)\right]^2}\, d\theta$

$= 2\int_0^{\pi} \cos\left(\frac{1}{2}\theta\right) d\theta = 4\left[\sin\left(\frac{1}{2}\theta\right)\right]_0^{\pi} = 4$

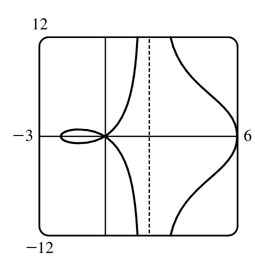
33. From Figure 4 in Example 1,

$L = \int_{-\pi/4}^{\pi/4} \sqrt{r^2 + (r')^2}\, d\theta$

$= 2\int_0^{\pi/4} \sqrt{\cos^2 2\theta + 4\sin^2 2\theta}\, d\theta$

$\approx 2\,(1.211056) \approx 2.4221$

34.



$4 + 2\sec\theta = 0 \quad\Rightarrow\quad \sec\theta = -2$

$\Rightarrow \quad \cos\theta = -\frac{1}{2} \quad\Rightarrow\quad \theta = \frac{2\pi}{3}, \frac{4\pi}{3}$.

$L = \int_{2\pi/3}^{4\pi/3} \sqrt{(4 + 2\sec\theta)^2 + (2\sec\theta\tan\theta)^2} d\theta \approx 5.8128$