9.5 CONIC SECTIONS IN POLAR COORDINATES

A Click here for answers.

S Click here for solutions.

1–8 ■ Write a polar equation of a conic with the focus at the origin and the given data.

1. Ellipse, eccentricity $\frac{2}{3}$, directrix $x = 3$

2. Hyperbola, eccentricity $\frac{4}{3}$, directrix $x = -3$

3. Parabola, directrix $y = 2$

4. Ellipse, eccentricity $\frac{1}{2}$, directrix $y = -4$

5. Hyperbola, eccentricity 4, directrix $r = 5 \sec \theta$

6. Ellipse, eccentricity 0.6, directrix $r = 2 \csc \theta$

7. Parabola, vertex at $(5, \pi/2)$

8. Ellipse, eccentricity 0.4, vertex at $(2, 0)$

9–16 ■ (a) Find the eccentricity, (b) identify the conic, (c) give an equation of the directrix, and (d) sketch the conic.

9. $r = \dfrac{4}{1 + 3 \cos \theta}$

10. $r = \dfrac{2}{1 - \cos \theta}$

11. $r = \dfrac{6}{2 + \sin \theta}$

12. $r = \dfrac{7}{2 - 5 \sin \theta}$

13. $r = \dfrac{8}{3 + 3 \cos \theta}$

14. $r = \dfrac{10}{3 - 2 \sin \theta}$

15. $r = \dfrac{5}{2 - 3 \sin \theta}$

16. $r = \dfrac{8}{3 + \cos \theta}$

9.5 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $r = \dfrac{6}{3 + 2\cos\theta}$

2. $r = \dfrac{12}{3 - 4\cos\theta}$

3. $r = \dfrac{2}{1 + \sin\theta}$

4. $r = \dfrac{4}{2 - \sin\theta}$

5. $r = \dfrac{20}{1 + 4\cos\theta}$

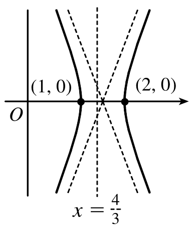
6. $r = \dfrac{6}{5 + 3\sin\theta}$

7. $r = \dfrac{10}{1 + \sin\theta}$

8. $r = \dfrac{8}{5 + 2\cos\theta}$

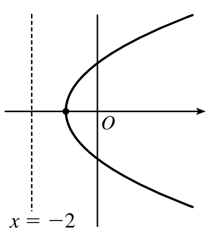
9. (a) 3 (b) Hyperbola (c) $x = \frac{4}{3}$

(d)



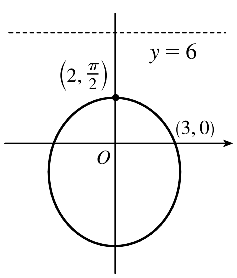
10. (a) 1 (b) Parabola (c) $x = -2$

(d)



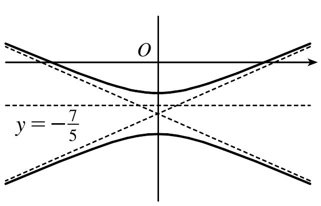
11. (a) $\frac{1}{2}$ (b) Ellipse (c) $y = 6$

(d)



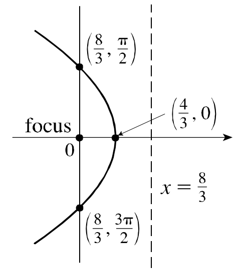
12. (a) $\frac{5}{2}$ (b) Hyperbola (c) $y = -\frac{7}{5}$

(d)



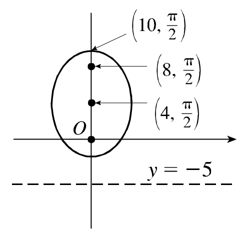
13. (a) 1 (b) Parabola (c) $x = \frac{8}{3}$

(d)



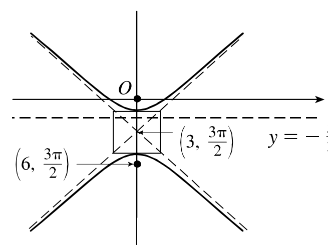
14. (a) $\frac{2}{3}$ (b) Ellipse (c) $y = -5$

(d)



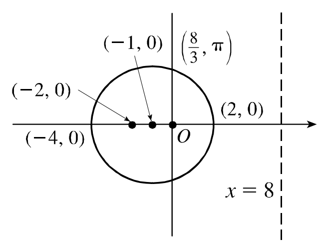
15. (a) $\frac{3}{2}$ (b) Hyperbola (c) $y = -\frac{5}{3}$

(d)



16. (a) $\frac{1}{3}$ (b) Ellipse (c) $x = 8$

(d)



9.5 SOLUTIONS

E Click here for exercises.

1. $r = \dfrac{ed}{1 + e\cos\theta} = \dfrac{\frac{2}{3} \cdot 3}{1 + \frac{2}{3}\cos\theta} = \dfrac{6}{3 + 2\cos\theta}$

2. $r = \dfrac{ed}{1 - e\cos\theta} = \dfrac{\frac{4}{3} \cdot 3}{1 - \frac{4}{3}\cos\theta} = \dfrac{12}{3 - 4\cos\theta}$

3. $r = \dfrac{ed}{1 + e\sin\theta} = \dfrac{1 \cdot 2}{1 + \sin\theta} = \dfrac{2}{1 + \sin\theta}$

4. $r = \dfrac{ed}{1 - e\sin\theta} = \dfrac{\frac{1}{2} \cdot 4}{1 - \frac{1}{2}\sin\theta} = \dfrac{4}{2 - \sin\theta}$

5. $r = 5\sec\theta \quad \Leftrightarrow \quad x = r\cos\theta = 5$, so

$$r = \frac{ed}{1 + e\cos\theta} = \frac{4 \cdot 5}{1 + 4\cos\theta} = \frac{20}{1 + 4\cos\theta}$$

6. $r = 2\csc\theta \quad \Leftrightarrow \quad y = r\sin\theta = 2$, so

$$r = \frac{ed}{1 + e\sin\theta} = \frac{\frac{3}{5} \cdot 2}{1 + \frac{3}{5}\sin\theta} = \frac{6}{5 + 3\sin\theta}$$

7. Focus $(0, 0)$, vertex $\left(5, \frac{\pi}{2}\right) \quad \Rightarrow \quad$ directrix $y = 10 \quad \Rightarrow$

$$r = \frac{ed}{1 + e\sin\theta} = \frac{10}{1 + \sin\theta}$$

8. The directrix is $x = 4$, so

$$r = \frac{ed}{1 + e\cos\theta} = \frac{\frac{2}{5} \cdot 4}{1 + \frac{2}{5}\cos\theta} = \frac{8}{5 + 2\cos\theta}$$

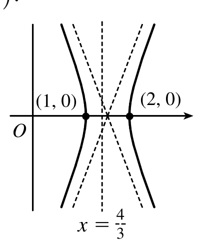
9. $r = \dfrac{4}{1 + 3\cos\theta}$

(a) $e = 3$

(b) Since $e = 3 > 1$, the conic is a hyperbola.

(c) $ed = 4 \quad \Rightarrow \quad d = \frac{4}{3} \quad \Rightarrow \quad$ directrix $x = \frac{4}{3}$

(d) The vertices are $(1, 0)$ and $(-2, \pi) = (2, 0)$; the center is $\left(\frac{3}{2}, 0\right)$; the asymptotes are parallel to $\theta = \pm\cos^{-1}\left(-\frac{1}{3}\right)$.



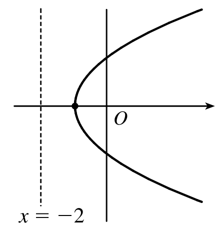
10. $r = \dfrac{2}{1 - \cos\theta}$

(a) $e = 1$

(b) Parabola

(c) $ed = 2 \quad \Rightarrow \quad d = 2 \quad \Rightarrow \quad$ directrix $x = -2$

(d) Vertex $(-1, 0) = (1, \pi)$



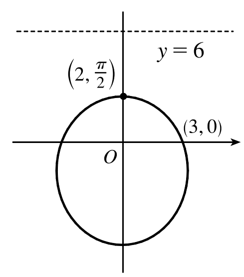
11. $r = \dfrac{3}{1 + \frac{1}{2}\sin\theta}$

(a) $e = \frac{1}{2}$

(b) Ellipse

(c) $ed = 3 \quad \Rightarrow \quad d = 6 \quad \Rightarrow \quad$ directrix $y = 6$

(d) Vertices $\left(2, \frac{\pi}{2}\right)$ and $\left(6, \frac{3\pi}{2}\right)$; center $\left(2, \frac{3\pi}{2}\right)$



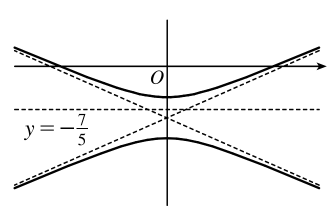
12. $r = \dfrac{7/2}{1 - \frac{5}{2}\sin\theta}$

(a) $e = \frac{5}{2}$

(b) Hyperbola

(c) $ed = \frac{7}{2} \quad \Rightarrow \quad d = \frac{7}{5} \quad \Rightarrow \quad$ directrix $y = -\frac{7}{5}$

(d) Center $\left(\frac{5}{3}, \frac{3\pi}{2}\right)$; vertices $\left(-\frac{7}{3}, \frac{\pi}{2}\right) = \left(\frac{7}{3}, \frac{3\pi}{2}\right)$ and $\left(1, \frac{3\pi}{2}\right)$



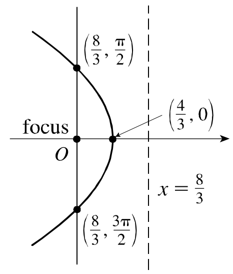
13. $r = \dfrac{8/3}{1+\cos\theta}$

(a) $e = 1$

(b) Parabola

(c) $ed = \frac{8}{3} \quad\Rightarrow\quad d = \frac{8}{3} \quad\Rightarrow\quad$ directrix $x = \frac{8}{3}$

(d) Vertex $\left(\frac{4}{3}, 0\right)$



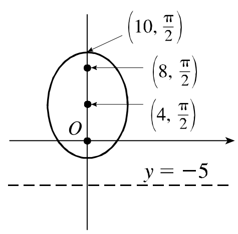
14. $r = \dfrac{10/3}{1-\frac{2}{3}\sin\theta}$

(a) $e = \frac{2}{3}$

(b) Ellipse

(c) $ed = \frac{10}{3} \quad\Rightarrow\quad d = 5 \quad\Rightarrow\quad$ directrix $y = -5$

(d) Vertices $\left(10, \frac{\pi}{2}\right)$ and $\left(2, \frac{3\pi}{2}\right)$; center $\left(4, \frac{\pi}{2}\right)$



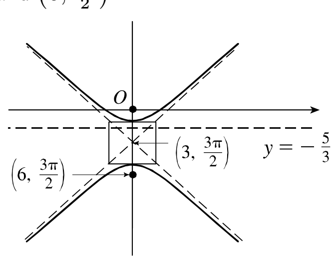
15. $r = \dfrac{5/2}{1-\frac{3}{2}\sin\theta}$

(a) $e = \frac{3}{2}$

(b) Hyperbola

(c) $ed = \frac{5}{2} \quad\Rightarrow\quad d = \frac{5}{3} \quad\Rightarrow\quad$ directrix $y = -\frac{5}{3}$

(d) Vertices $\left(-5, \frac{\pi}{2}\right) = \left(5, \frac{3\pi}{2}\right)$ and $\left(1, \frac{3\pi}{2}\right)$; center $\left(3, \frac{3\pi}{2}\right)$; foci $(0, 0)$ and $\left(6, \frac{3\pi}{2}\right)$



16. $r = \dfrac{8/3}{1+\frac{1}{3}\cos\theta}$

(a) $e = \frac{1}{3}$

(b) Ellipse

(c) $ed = \frac{8}{3} \quad\Rightarrow\quad d = 8 \quad\Rightarrow\quad$ directrix $x = 8$

(d) Vertices $(2, 0)$ and $(4, \pi)$; center $(-1, 0)$