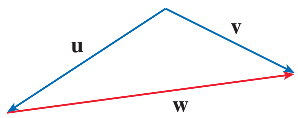
10.2 VECTORS

[A] Click here for answers.

[S] Click here for solutions.

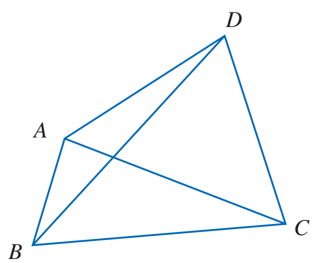
1. Express **w** in terms of the vectors **u** and **v** in the figure.



2. Write each combination of vectors as a single vector.

(a) $\overrightarrow{AB} + \overrightarrow{BC}$ (b) $\overrightarrow{CD} + \overrightarrow{DA}$

(c) $\overrightarrow{BC} - \overrightarrow{DC}$ (d) $\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA}$



3–5 ▪ Find a vector **a** with representation given by the directed line segment $\overrightarrow{AB}$. Draw $\overrightarrow{AB}$ and the equivalent representation starting at the origin.

3. $A(1, 3)$, $B(4, 4)$

4. $A(4, -1)$, $B(1, 2)$

5. $A(1, -2, 0)$, $B(1, -2, 3)$

6–9 ▪ Find the sum of the given vectors and illustrate geometrically.

6. $\langle 2, 3 \rangle$, $\langle 3, -4 \rangle$

7. $\langle -1, 2 \rangle$, $\langle 5, 3 \rangle$

8. $\langle 1, 0, 1 \rangle$, $\langle 0, 0, 1 \rangle$

9. $\langle 0, 3, 2 \rangle$, $\langle 1, 0, -3 \rangle$

10–15 ▪ Find a unit vector that has the same direction as the given vector.

10. $\langle 1, 2 \rangle$

11. $\langle 3, -5 \rangle$

12. $\langle -2, 4, 3 \rangle$

13. $\langle 1, -4, 8 \rangle$

14. $\mathbf{i} + \mathbf{j}$

15. $2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$

16. A quadrilateral has one pair of opposite sides parallel and of equal length. Use vectors to prove that the other pair of opposite sides is parallel and of equal length.

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10.2 ANSWERS

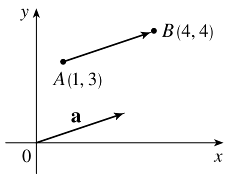
E Click here for exercises.

S Click here for solutions.

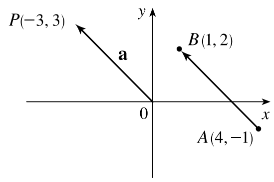
1. $\mathbf{w} = \mathbf{v} - \mathbf{u}$

2. (a) $\overrightarrow{AC}$ (b) $\overrightarrow{CA}$ (c) $\overrightarrow{BD}$ (d) $\overrightarrow{BA}$

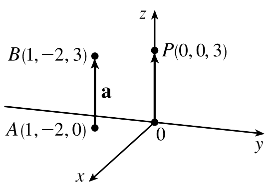
3. $\langle 3, 1 \rangle$



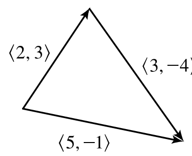
4. $\langle -3, 3 \rangle$



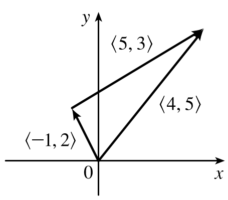
5. $\langle 0, 0, 3 \rangle$



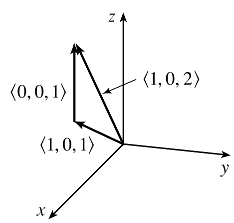
6. $\langle 5, -1 \rangle$



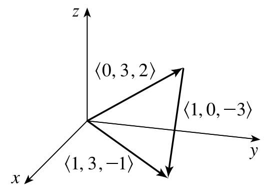
7. $\langle 4, 5 \rangle$



8. $\langle 1, 0, 2 \rangle$



9. $\langle 1, 3, -1 \rangle$



10. $\left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$

11. $\left\langle \frac{3}{\sqrt{34}}, -\frac{5}{\sqrt{34}} \right\rangle$

12. $\left\langle -\frac{2}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{3}{\sqrt{29}} \right\rangle$

13. $\left\langle \frac{1}{9}, -\frac{4}{9}, \frac{8}{9} \right\rangle$

14. $\frac{1}{\sqrt{2}}\,\mathbf{i} + \frac{1}{\sqrt{2}}\,\mathbf{j}$

15. $\frac{2}{\sqrt{69}}\,\mathbf{i} - \frac{4}{\sqrt{69}}\,\mathbf{j} + \frac{7}{\sqrt{69}}\,\mathbf{k}$

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10.2 SOLUTIONS

E Click here for exercises.

1. Geometrically, by the Triangle Law, we can see that $\mathbf{u} + \mathbf{w} = \mathbf{v}$, thus $\mathbf{w} = \mathbf{v} - \mathbf{u}$. Alternately, $\mathbf{w}$ can be visualized directly as the difference of $\mathbf{v}$ and $\mathbf{u}$ (see Figure 8 in the text).

2. (a) By the Triangle Law, $\overrightarrow{AB} + \overrightarrow{BC}$ is the vector with initial point A and terminal point C, namely $\overrightarrow{AC}$.

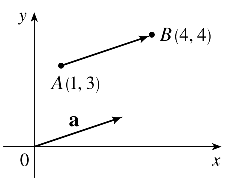
(b) By the Triangle Law, $\overrightarrow{CD} + \overrightarrow{DA}$ is the vector with initial point C and terminal point A, namely $\overrightarrow{CA}$.

(c) First we consider $\overrightarrow{BC} - \overrightarrow{DC}$ as $\overrightarrow{BC} + \left(-\overrightarrow{DC}\right)$. Then since $-\overrightarrow{DC}$ has the same length as $\overrightarrow{CD}$ but points in the opposite direction, we have $-\overrightarrow{DC} = \overrightarrow{CD}$ and so $\overrightarrow{BC} - \overrightarrow{DC} = \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{BD}$.

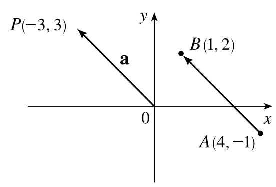
(d) We use the Triangle Law twice:

$$\begin{aligned}\overrightarrow{BC} + \overrightarrow{CD} + \overrightarrow{DA} &= \left(\overrightarrow{BC} + \overrightarrow{CD}\right) + \overrightarrow{DA} \\ &= \overrightarrow{BD} + \overrightarrow{DA} = \overrightarrow{BA}\end{aligned}$$

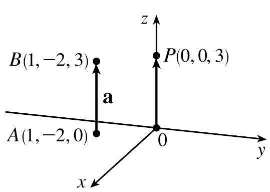
3. $\mathbf{a} = \langle 4-1, 4-3\rangle = \langle 3, 1\rangle$



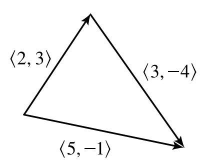
4. $\mathbf{a} = \langle 1-4, 2+1\rangle$
$= \langle -3, 3\rangle$



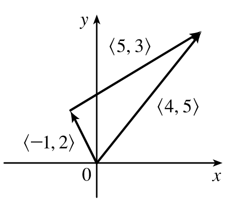
5. $\mathbf{a} = \langle 1-1, -2+2, 3-0\rangle$
$= \langle 0, 0, 3\rangle$



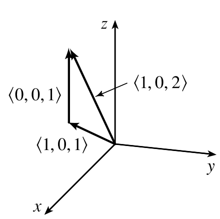
6. $\langle 2, 3\rangle + \langle 3, -4\rangle = \langle 5, -1\rangle$
(using position vectors and the parallelogram law)



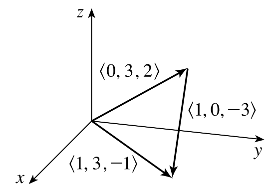
7. $\langle -1, 2\rangle + \langle 5, 3\rangle$
$= \langle -1+5, 2+3\rangle$
$= \langle 4, 5\rangle$



8. $\langle 1, 0, 1\rangle + \langle 0, 0, 1\rangle$
$= \langle 1+0, 0+0, 1+1\rangle$
$= \langle 1, 0, 2\rangle$



9. $\langle 0, 3, 2\rangle + \langle 1, 0, -3\rangle = \langle 1, 3, -1\rangle$



10. $|\langle 1, 2\rangle| = \sqrt{1^2 + 2^2} = \sqrt{5}$. Thus $\mathbf{u} = \frac{1}{\sqrt{5}}\langle 1, 2\rangle = \left\langle \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right\rangle$.

11. $|\langle 3, -5\rangle| = \sqrt{3^2 + (-5)^2} = \sqrt{34}$. Thus $\mathbf{u} = \frac{1}{\sqrt{34}}\langle 3, -5\rangle = \left\langle \frac{3}{\sqrt{34}}, -\frac{5}{\sqrt{34}}\right\rangle$.

12. $|\langle -2, 4, 3\rangle| = \sqrt{(-2)^2 + 4^2 + 3^2} = \sqrt{29}$. Thus $\mathbf{u} = \frac{1}{\sqrt{29}}\langle -2, 4, 3\rangle = \left\langle -\frac{2}{\sqrt{29}}, \frac{4}{\sqrt{29}}, \frac{3}{\sqrt{29}}\right\rangle$.

13. $|\langle 1, -4, 8\rangle| = \sqrt{1^2 + (-4)^2 + 8^2} = \sqrt{81} = 9$. Thus $\mathbf{u} = \frac{1}{9}\langle 1, -4, 8\rangle = \left\langle \frac{1}{9}, -\frac{4}{9}, \frac{8}{9}\right\rangle$.

14. $|\mathbf{i} + \mathbf{j}| = \sqrt{1^2 + 1^2} = \sqrt{2}$. Thus $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) = \frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}$.

15. $|2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}| = \sqrt{2^2 + (-4)^2 + 7^2} = \sqrt{69}$. Thus $\mathbf{u} = \frac{1}{\sqrt{69}}(2\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}) = \frac{2}{\sqrt{69}}\mathbf{i} - \frac{4}{\sqrt{69}}\mathbf{j} + \frac{7}{\sqrt{69}}\mathbf{k}$.

16. Consider quadrilateral $ABCD$ with sides AB and CD parallel and of equal length; that is, $\overrightarrow{AB} = \overrightarrow{DC}$. Thus

$$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{DC} + \overrightarrow{BD} \quad \left(\text{since } \overrightarrow{AB} = \overrightarrow{DC}\right) \\ &= \overrightarrow{BD} + \overrightarrow{DC} = \overrightarrow{BC}\end{aligned}$$

This shows that sides AD and BC are parallel and have equal lengths.

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