10.5 EQUATIONS OF LINES AND PLANES

A Click here for answers.

S Click here for solutions.

1–4 ▪ Find a vector equation and parametric equations for the line passing through the given point and parallel to the vector **a**.

1. $(3, -1, 8)$, $\mathbf{a} = \langle 2, 3, 5 \rangle$

2. $(-2, 4, 5)$, $\mathbf{a} = \langle 3, -1, 6 \rangle$

3. $(0, 1, 2)$, $\mathbf{a} = 6\,\mathbf{i} + 3\,\mathbf{j} + 2\,\mathbf{k}$

4. $(1, -1, -2)$, $\mathbf{a} = 2\,\mathbf{i} - 7\,\mathbf{k}$

5–10 ▪ Find parametric equations and symmetric equations for the line through the given points.

5. $(2, 1, 8)$, $(6, 0, 3)$

6. $(-1, 0, 5)$, $(4, -3, 3)$

7. $(3, 1, -1)$, $(3, 2, -6)$

8. $\left(3, 1, \frac{1}{2}\right)$, $(-1, 4, 1)$

9. $\left(-\frac{1}{3}, 1, 1\right)$, $(0, 5, -8)$

10. $(2, -7, 5)$, $(-4, 2, 5)$

11. Show that the line through the points $(0, 1, 1)$ and $(1, -1, 6)$ is perpendicular to the line through the points $(-4, 2, 1)$ and $(-1, 6, 2)$.

12–14 ▪ Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

12. L_1: $\dfrac{x-4}{2} = \dfrac{y+5}{4} = \dfrac{z-1}{-3}$,

L_2: $\dfrac{x-2}{1} = \dfrac{y+1}{3} = \dfrac{z}{2}$

13. L_1: $\dfrac{x-1}{2} = \dfrac{y}{1} = \dfrac{z-1}{4}$,

L_2: $\dfrac{x}{1} = \dfrac{y+2}{2} = \dfrac{z+2}{3}$

14. L_1: $x = 1 + t$, $y = 2 - t$, $z = 3t$

L_2: $x = 2 - s$, $y = 1 + 2s$, $z = 4 + s$

15–18 ▪ Find an equation of the plane passing through the given point and with normal vector **n**.

15. $(1, 4, 5)$, $\mathbf{n} = \langle 7, 1, 4 \rangle$

16. $(-5, 1, 2)$, $\mathbf{n} = \langle 3, -5, 2 \rangle$

17. $(1, 2, 3)$, $\mathbf{n} = 15\,\mathbf{i} + 9\,\mathbf{j} - 12\,\mathbf{k}$

18. $(-1, -6, -4)$, $\mathbf{n} = -5\,\mathbf{i} + 2\,\mathbf{j} - 2\,\mathbf{k}$

19–22 ▪ Find an equation of the plane passing through the given point and parallel to the specified plane.

19. $(6, 5, -2)$, $x + y - z + 1 = 0$

20. $(3, 0, 8)$, $2x + 5y + 8z = 17$

21. $(-1, 3, -8)$, $3x - 4y - 6z = 9$

22. $(2, -4, 5)$, $z = 2x + 3y$

23–26 ▪ Find an equation of the plane passing through the three given points.

23. $(0, 0, 0)$, $(1, 1, 1)$, $(1, 2, 3)$

24. $(-1, 1, -1)$, $(1, -1, 2)$, $(4, 0, 3)$

25. $(1, 0, -3)$, $(0, -2, -4)$, $(4, 1, 6)$

26. $(2, 1, -3)$, $(5, -1, 4)$, $(2, -2, 4)$

27–30 ▪ Find an equation of the plane that passes through the given point and contains the specified line.

27. $(1, 6, -4)$; $x = 1 + 2t$, $y = 2 - 3t$, $z = 3 - t$

28. $(-1, -3, 2)$; $x = -1 - 2t$, $y = 4t$, $z = 2 + t$

29. $(0, 1, 2)$; $x = y = z$

30. $(-1, 0, 1)$; $x = 5t$, $y = 1 + t$, $z = -t$

31–34 ▪ Find the point at which the line intersects the given plane.

31. $x = 1 + t$, $y = 2t$, $z = 3t$; $x + y + z = 1$

32. $x = 5$, $y = 4 - t$, $z = 2t$; $2x - y + z = 5$

33. $x = 1 + 2t$, $y = -1$, $z = t$; $2x + y - z + 5 = 0$

34. $x = 1 - t$, $y = t$, $z = 1 + t$; $z = 1 - 2x + y$

35–40 ▪ Determine whether the planes are parallel, perpendicular, or neither. If neither, find the angle between them.

35. $x + z = 1$, $y + z = 1$

36. $-8x - 6y + 2z = 1$, $z = 4x + 3y$

37. $x + 4y - 3z = 1$, $-3x + 6y + 7z = 0$

38. $2x + 2y - z = 4$, $6x - 3y + 2z = 5$

39. $2x + 4y - 2z = 1$, $-3x - 6y + 3z = 10$

40. $2x - 5y + z = 3$, $4x + 2y + 2z = 1$

10.5 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $\mathbf{r} = (3 + 2t)\,\mathbf{i} + (-1 + 3t)\,\mathbf{j} + (8 + 5t)\,\mathbf{k}$;
$x = 3 + 2t,\ y = -1 + 3t,\ z = 8 + 5t$

2. $\mathbf{r} = (-2 + 3t)\,\mathbf{i} + (4 - t)\,\mathbf{j} + (5 + 6t)\,\mathbf{k}$;
$x = -2 + 3t,\ y = 4 - t,\ z = 5 + 6t$

3. $\mathbf{r} = (6t)\,\mathbf{i} + (1 + 3t)\,\mathbf{j} + (2 + 2t)\,\mathbf{k}$;
$x = 6t,\ y = 1 + 3t,\ z = 2 + 2t$

4. $\mathbf{r} = (1 + 2t)\,\mathbf{i} - \mathbf{j} + (-2 - 7t)\,\mathbf{k}$;
$x = 1 + 2t,\ y = -1,\ z = -(2 + 7t)$

5. $x = 2 + 4t,\ y = 1 - t,\ z = 8 - 5t$;
$\dfrac{x-2}{4} = \dfrac{y-1}{-1} = \dfrac{z-8}{-5}$

6. $x = -1 + 5t,\ y = -3t,\ z = 5 - 2t$; $\dfrac{x+1}{5} = \dfrac{y}{-3} = \dfrac{z-5}{-2}$

7. $x = 3,\ y = 1 + t,\ z = -1 - 5t$; $x = 3,\ y - 1 = \dfrac{z+1}{-5}$

8. $x = -1 - 4t,\ y = 4 + 3t,\ z = 1 + \frac{1}{2}t$;
$\dfrac{x+1}{-4} = \dfrac{y-4}{3} = \dfrac{z-1}{1/2}$

9. $x = -\frac{1}{3} + \frac{1}{3}t,\ y = 1 + 4t,\ z = 1 - 9t$;
$\dfrac{x+1/3}{1/3} = \dfrac{y-1}{4} = \dfrac{z-1}{-9}$

10. $x = 2 - 6t,\ y = -7 + 9t,\ z = 5$; $\dfrac{x-2}{-6} = \dfrac{y+7}{9},\ z = 5$

12. Skew

13. Intersecting, $(1, 0, 1)$

14. Skew

15. $7x + y + 4z = 31$

16. $3x - 5y + 2z = -16$

17. $5x + 3y - 4z = -1$

18. $-5x + 2y - 2z = 1$

19. $x + y - z = 13$

20. $2x + 5y + 8z = 70$

21. $3x - 4y - 6z = 33$

22. $2x + 3y - z = -13$

23. $x - 2y + z = 0$

24. $-5x + 7y + 8z = 4$

25. $-17x + 6y + 5z = -32$

26. $7x - 21y - 9z = 20$

27. $25x + 14y + 8z = 77$

28. $x + 2z = 3$

29. $x - 2y + z = 0$

30. $y + z = 1$

31. $(1, 0, 0)$

32. $\left(5, \frac{13}{3}, -\frac{2}{3}\right)$

33. $(-3, -1, -2)$

34. $(0, 1, 2)$

35. Neither, 60°

36. Parallel

37. Perpendicular

38. Neither, 79°

39. Parallel

40. Perpendicular

10.5 SOLUTIONS

E Click here for exercises.

1. $\mathbf{r}_0 = 3\,\mathbf{i} - \mathbf{j} + 8\,\mathbf{k}$ and $\mathbf{v} = \mathbf{a}$ so a vector equation is

$$\mathbf{r} = (3\,\mathbf{i} - \mathbf{j} + 8\,\mathbf{k}) + t\,(2\,\mathbf{i} + 3\,\mathbf{j} + 5\,\mathbf{k}) = (3 + 2t)\,\mathbf{i} + (-1 + 3t)\,\mathbf{j} + (8 + 5t)\,\mathbf{k}$$

and parametric equations are $x = 3 + 2t$, $y = -1 + 3t$, $z = 8 + 5t$.

2. $\mathbf{r}_0 = -2\,\mathbf{i} + 4\,\mathbf{j} + 5\,\mathbf{k}$, $\mathbf{v} = \mathbf{a}$ so a vector equation is

$$\mathbf{r} = (-2\,\mathbf{i} + 4\,\mathbf{j} + 5\,\mathbf{k}) + t\,(3\,\mathbf{i} - \mathbf{j} + 6\,\mathbf{k}) = (-2 + 3t)\,\mathbf{i} + (4 - t)\,\mathbf{j} + (5 + 6t)\,\mathbf{k}$$

while parametric equations are: $x = -2 + 3t$, $y = 4 - t$, $z = 5 + 6t$.

3. $$\mathbf{r} = (\mathbf{j} + 2\,\mathbf{k}) + t\,(6\,\mathbf{i} + 3\,\mathbf{j} + 2\,\mathbf{k}) = (6t)\,\mathbf{i} + (1 + 3t)\,\mathbf{j} + (2 + 2t)\,\mathbf{k}$$

is a vector equation, while $x = 6t$, $y = 1 + 3t$, $z = 2 + 2t$ are parametric equations.

4. $$\mathbf{r} = (\mathbf{i} - \mathbf{j} - 2\,\mathbf{k}) + t\,(2\,\mathbf{i} - 7\,\mathbf{k}) = (1 + 2t)\,\mathbf{i} - \mathbf{j} + (-2 - 7t)\,\mathbf{k}$$

is a vector equation, while $x = 1 + 2t$, $y = -1$, $z = -(2 + 7t)$ are parametric equations.

5. The parallel vector is $\mathbf{v} = \langle 6 - 2, 0 - 1, 3 - 8\rangle = \langle 4, -1, -5\rangle$ so the direction numbers are $a = 4$, $b = -1$, $c = -5$. Letting $P_0 = (2, 1, 8)$, parametric equations are $x = 2 + 4t$, $y = 1 - t$, $z = 8 - 5t$, and symmetric equations are $\dfrac{x - 2}{4} = \dfrac{y - 1}{-1} = \dfrac{z - 8}{-5}$.

6. $\mathbf{v} = \langle 5, -3, -2\rangle$ and letting $P_0 = (-1, 0, 5)$, parametric equations are $x = -1 + 5t$, $y = -3t$, $z = 5 - 2t$ while symmetric equations are $\dfrac{x + 1}{5} = \dfrac{y}{-3} = \dfrac{z - 5}{-2}$.

7. $\mathbf{v} = \langle 0, 1, -5\rangle$ and letting $P_0 = (3, 1, -1)$, parametric equations are $x = 3$, $y = 1 + t$, $z = -1 - 5t$ while symmetric equations are $x = 3$, $y - 1 = \dfrac{z + 1}{-5}$. Notice here that the direction number $a = 0$, so rather than writing $\dfrac{x - 3}{0}$ in the symmetric equation we write the equation $x = 3$ separately.

8. $\mathbf{v} = \left\langle -4, 3, \frac{1}{2}\right\rangle$ and letting $P_0 = (-1, 4, 1)$, parametric equations are $x = -1 - 4t$, $y = 4 + 3t$, $z = 1 + \frac{1}{2}t$ while symmetric equations are $\dfrac{x + 1}{-4} = \dfrac{y - 4}{3} = \dfrac{z - 1}{1/2}$.

9. $\mathbf{v} = \left\langle \frac{1}{3}, 4, -9\right\rangle$ and letting $P_0 = \left(-\frac{1}{3}, 1, 1\right)$, parametric equations are $x = -\frac{1}{3} + \frac{1}{3}t$, $y = 1 + 4t$, $z = 1 - 9t$ while symmetric equations are $\dfrac{x + 1/3}{1/3} = \dfrac{y - 1}{4} = \dfrac{z - 1}{-9}$.

10. $\mathbf{v} = \langle -6, 9, 0\rangle$ and letting $P_0 = (2, -7, 5)$, parametric equations are $x = 2 - 6t$, $y = -7 + 9t$, $z = 5$ while symmetric equations are $\dfrac{x - 2}{-6} = \dfrac{y + 7}{9}$, $z = 5$.

11. Direction vectors of the lines are $\mathbf{v}_1 = \langle 1, -2, 5\rangle$ and $\mathbf{v}_2 = \langle 3, 4, 1\rangle$. Since $\mathbf{v}_1 \cdot \mathbf{v}_2 = 3 - 8 + 5 = 0$, the direction vectors and thus the lines are perpendicular.

12. The lines aren't parallel since the direction vectors $\langle 2, 4, -3\rangle$ and $\langle 1, 3, 2\rangle$ aren't parallel, so we check to see if the lines intersect. The parametric equations of the lines are L_1: $x = 4 + 2t$, $y = -5 + 4t$, $z = 1 - 3t$ and L_2: $x = 2 + s$, $y = -1 + 3s$, $z = 2s$. For the lines to intersect we must be able to find one value of t and one value of s satisfying the following three equations: $4 + 2t = 2 + s$, $-5 + 4t = -1 + 3s$, $1 - 3t = 2s$. Solving the first two equations we get $t = -5$, $s = -8$ and checking, we see that these values don't satisfy the third equation. Thus L_1 and L_2 aren't parallel and don't intersect, so they must be skew lines.

13. Since the direction vectors $\langle 2, 1, 4\rangle$ and $\langle 1, 2, 3\rangle$ aren't parallel, the lines aren't parallel. Here the parametric equations are L_1: $x = 1 + 2t$, $y = t$, $z = 1 + 4t$; L_2: $x = s$, $y = -2 + 2s$, $z = -2 + 3s$. Thus, for the lines to intersect, the three equations $1 + 2t = s$, $t = -2 + 2s$ and $1 + 4t = -2 + 3s$ must be satisfied simultaneously. Solving the first two equations gives $t = 0$, $s = 1$ and, checking, we see these values do satisfy the third equation, so the lines intersect when $t = 0$ and $s = 1$, that is, at the point $(1, 0, 1)$.

14. Since the direction vectors are $\langle 1, -1, 3\rangle$ and $\langle -1, 2, 1\rangle$, the lines aren't parallel. For the lines to intersect, the three equations $1 + t = 2 - s$, $2 - t = 1 + 2s$, $3t = 4 + s$ must be satisfied simultaneously. Solving the first two equations gives $t = 1$, $s = 0$ and, checking, we see these values don't satisfy the third equation. Thus L_1 and L_2 aren't parallel and don't intersect, so they must be skew lines.

15. Setting $a = 7$, $b = 1$, $c = 4$, $x_0 = 1$, $y_0 = 4$, $z_0 = 5$ in Equation 7 gives $7\,(x - 1) + 1\,(y - 4) + 4\,(z - 5) = 0$ or $7x + y + 4z = 31$ to be an equation of the plane.

16. Setting $a = 3$, $b = -5$, $c = 2$, $x_0 = -5$, $y_0 = 1$, $z_0 = 2$ in Equation 7 gives $3\,(x + 5) - 5\,(y - 1) + 2\,(z - 2) = 0$ or $3x - 5y + 2z = -16$ to be an equation of the plane.

17. Setting $a = 15$, $b = 9$, $c = -12$, $x_0 = 1$, $y_0 = 2$, $z_0 = 3$ in Equation 7 gives $15\,(x - 1) + 9\,(y - 2) - 12\,(z - 3) = 0$ or $5x + 3y - 4z = -1$ to be an equation of the plane.

18. Setting $a = -5$, $b = 2$, $c = -2$, $x_0 = -1$, $y_0 = -6$, $z_0 = -4$ in Equation 7 gives $-5(x+1) + 2(y+6) - 2(z+4) = 0$ or $-5x + 2y - 2z = 1$ to be an equation of the plane.

19. Since the two planes are parallel, they have the same normal vector. Thus $\mathbf{n} = \langle 1, 1, -1\rangle$ and an equation of the plane is $1(x-6) + 1(y-5) + 1(z-2) = 0$ or $x + y - z = 13$.

20. Since the two planes are parallel, they have the same normal vector. Thus $\mathbf{n} = \langle 2, 5, 8\rangle$ and an equation of the plane is $2(x-3) + 5(y-0) + 8(z-8) = 0$ or $2x + 5y + 8z = 70$.

21. An equation is $3(x+1) - 4(y-3) - 6(z+8) = 0$ or $3x - 4y - 6z = 33$.

22. An equation is $2(x-2) + 3(y+4) - (z-5) = 0$ or $2x + 3y - z = -13$.

23. Here the vectors $\mathbf{a} = \langle 1, 1, 1\rangle$ and $\mathbf{b} = \langle 1, 2, 3\rangle$ lie in the plane, so $\mathbf{a} \times \mathbf{b}$ is a normal vector to the plane. Thus $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 3-2, 1-3, 2-1\rangle = \langle 1, -2, 1\rangle$ and an equation of the plane is $x - 2y + z = 0$.

24. Here the vectors $\mathbf{a} = \langle 2, -2, 3\rangle$ and $\mathbf{b} = \langle 5, -1, 4\rangle$ lie in the plane, so a normal vector to the plane is $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -8+3, 15-8, -2+10\rangle = \langle -5, 7, 8\rangle$ and an equation of the plane is $-5(x+1) + 7(y-1) + 8(z+1) = 0$ or $-5x + 7y + 8z = 4$.

25. $\mathbf{a} = \langle -1, -2, -1\rangle$ and $\mathbf{b} = \langle 3, 1, 9\rangle$ so a normal vector to the plane is $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -18+1, -3+9, -1+6\rangle = \langle -17, 6, 5\rangle$ and an equation of the plane is $-17(x-1) + 6(y-0) + 5(z+3) = 0$ or $-17x + 6y + 5z = -32$.

26. $\mathbf{a} = \langle 3, -2, 7\rangle$ and $\mathbf{b} = \langle -3, -1, 0\rangle$ so a normal vector to the plane is $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 0+7, -21-0, -3-6\rangle = \langle 7, -21, -9\rangle$ and an equation of the plane is $7(x-2) - 21(y-1) - 9(z+3) = 0$ or $7x - 21y - 9z = 20$.

27. To find an equation of the plane we must first find two nonparallel vectors in the plane, then their cross product will be a normal vector to the plane. But since the given line lies in the plane, its direction vector $\mathbf{a} = \langle 2, -3, -1\rangle$ is one vector in the plane. To find another nonparallel vector $\mathbf{b}$ which lies in the plane pick any point on the line [say $(1, 2, 3)$, found by setting $t = 0$] and let $\mathbf{b}$ be the vector connecting this point to the given point in the plane. (But beware; we should first check that the given point is not on the given line. If it were on the line, the plane wouldn't be uniquely determined. What would $\mathbf{n}$ then be when we set $\mathbf{n} = \mathbf{a} \times \mathbf{b}$?) Here $\mathbf{b} = \langle 0, 4, -7\rangle$ so $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 21+4, 0+14, 8-0\rangle = \langle 25, 14, 8\rangle$ and an equation of the plane is $25(x-1) + 14(y-7) + 8(z+4) = 0$ or $25x + 14y + 8z = 77$.

28. As in Problem 27, set $\mathbf{a} = \langle -2, 4, 1\rangle$ and let $\mathbf{b}$ be the vector connecting, say $(-1, 0, 2)$ to $(-1, -3, 2)$. Thus $\mathbf{b} = \langle 0, -3, 0\rangle$ and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 0+3, 0-0, 6-0\rangle = \langle 3, 0, 6\rangle$ and an equation of the plane is $3(x+1) + 6(z-2) = 0$ or $x + 2z = 3$.

29. $(0, 0, 0)$ is a point on $x = y = z$. $\langle 1, 1, 1\rangle$ is the direction of this line, and thus also of the plane. $\langle 0-0, 1-0, 2-0\rangle = \langle 0, 1, 2\rangle$ is also a vector in the plane. Therefore,

$$\mathbf{n} = \langle 1, 1, 1\rangle \times \langle 0, 1, 2\rangle = \langle 2-1, -2+0, 1-0\rangle = \langle 1, -2, 1\rangle$$

Choosing $(x_0, y_0, z_0) = (0, 0, 0)$, an equation of the plane is, by Equation 8, $x - 2y + z = 1 \cdot 0 - 2 \cdot 0 + 1 \cdot 0 \Leftrightarrow x - 2y + z = 0$.

30. $\mathbf{a} = \langle 5, 1, -1\rangle$ is the direction of the line and hence also a direction of the plane. $(0, 1, 0)$ is a point on the line (and in the plane) so that $\mathbf{b} = \langle 0+1, 1-0, 0-1\rangle = \langle 1, 1, -1\rangle$ is another direction of the plane. $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -1+1, 5-1, 5-1\rangle = \langle 0, 4, 4\rangle$ and an equation of the plane is $0(x+1) + 4(y-0) + 4(z-1) = 0 \Leftrightarrow y + z = 1$.

31. Substituting the parametric equations of the line into the equation of the plane gives $x + y + z = 1 + t + 2t + 3t = 1 \Rightarrow t = 0$. This value of t corresponds to the point of intersection $(1, 0, 0)$, obtained by substitution of $t = 0$ into the equations of the line.

32. Substitute the parametric expressions for x, y and z into the equation of the plane: $2x - y + z = 2(5) - (4-t) + 2t = 5 \Rightarrow t = -\frac{1}{3}$. Therefore, the point of intersection of the line and the plane is given by $x = 5$, $y = 4 - \left(-\frac{1}{3}\right) = \frac{13}{3}$ and $z = 2\left(-\frac{1}{3}\right) = -\frac{2}{3}$, that is, the point $\left(5, \frac{13}{3}, -\frac{2}{3}\right)$.

33. Substituting the parametric equations of the line into the equation of the plane gives $2x + y - z + 5 = 2(1+2t) + (-1) - t + 5 = 0 \Rightarrow 3t + 6 = 0 \Rightarrow t = -2$. Therefore, the point of intersection is $x = 1 + 2(-2) = -3$, $y = -1$ and $z = -2$ and the point of intersection is $(-3, -1, -2)$.

34. Substitution into the equation of the plane of the parametric expressions for x, y and z gives $z = 1 - 2x + y \quad \Rightarrow$ $(1 + t) = 1 - 2(1 - t) + t \quad \Rightarrow \quad -2 + 2t = 0 \quad \Rightarrow$ $t = 1$. Thus, $x = 1 - 1$, $y = 1$ and $z = 1 + 1$ and the point of intersection is $(0, 1, 2)$.

35. The normal vectors to the planes are $\mathbf{n}_1 = \langle 1, 0, 1 \rangle$ and $\mathbf{n}_2 = \langle 0, 1, 1 \rangle$. Thus the normal vectors (and consequently the planes) aren't parallel. Furthermore, $\mathbf{n}_1 \cdot \mathbf{n}_2 = 1 \neq 0$ so the planes aren't perpendicular. Letting θ be the angle between the two planes, we have $\cos\theta = \dfrac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|} = \dfrac{1}{\sqrt{2}\sqrt{2}} = \dfrac{1}{2}$ and $\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$.

36. Here the normals are $\mathbf{n}_1 = \langle -8, -6, 2 \rangle$ and $\mathbf{n}_2 = \langle 4, 3, -1 \rangle$. Since $\mathbf{n}_1 = -2\mathbf{n}_2$, the normals (and thus the planes) are parallel.

37. The normals are $\mathbf{n}_1 = \langle 1, 4, -3 \rangle$ and $\mathbf{n}_2 = \langle -3, 6, 7 \rangle$, so the normals (and thus the planes) aren't parallel. But $\mathbf{n}_1 \cdot \mathbf{n}_2 = -3 + 24 - 21 = 0$, so the normals (and thus the planes) are perpendicular.

38. The normals are $\mathbf{n}_1 = \langle 2, 2, -1 \rangle$ and $\mathbf{n}_2 = \langle 6, -3, 2 \rangle$ so the planes aren't parallel. Furthermore, $\mathbf{n}_1 \cdot \mathbf{n}_2 = 12 - 6 - 2 = 4 \neq 0$, so the planes aren't perpendicular. Then $\cos\theta = \dfrac{4}{\sqrt{9}\sqrt{49}} = \dfrac{4}{21}$ and $\theta = \cos^{-1}\left(\frac{4}{21}\right) \approx 79^\circ$.

39. The normals are $\mathbf{n}_1 = \langle 2, 4, -2 \rangle$ and $\mathbf{n}_2 = \langle -3, -6, 3 \rangle$. Since $\mathbf{n}_1 = -\frac{3}{2}\mathbf{n}_2$, the normals (and thus the planes) are parallel.

40. The normals are $\mathbf{n}_1 = \langle 2, -5, 1 \rangle$ and $\mathbf{n}_2 = \langle 4, 2, 2 \rangle$. $\mathbf{n}_1 \cdot \mathbf{n}_2 = \langle 2, -5, 1 \rangle \cdot \langle 4, 2, 2 \rangle = 8 - 10 + 2 = 0$, so the normals (and thus the planes) are perpendicular.