10.6 CYLINDERS AND QUADRIC SURFACES

A Click here for answers.

S Click here for solutions.

1–5 ■ Find the traces of the given surface in the planes $x = k$, $y = k$, $z = k$. Then identify the surface and sketch it.

1. $x = y^2 + z^2$

2. $2x^2 + z^2 = 4$

3. $x^2 - y^2 + z^2 = 1$

4. $4z^2 - x^2 - y^2 = 1$

5. $9x^2 - y^2 - z^2 = 9$

6–13 ■ Reduce the equation to one of the standard forms, classify the surface, and sketch it.

6. $z^2 = 3x^2 + 4y^2 - 12$

7. $4x^2 - 9y^2 + z^2 + 36 = 0$

8. $z = x^2 + y^2 + 1$

9. $x^2 + 4y^2 + z^2 - 2x = 0$

10. $x^2 + y^2 - 4z^2 + 4x - 6y - 8z = 13$

11. $4x = y^2 - 2z^2$

12. $x^2 - y^2 + 4y + z = 4$

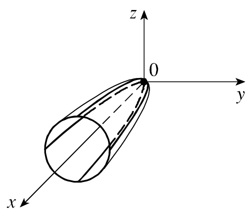
13. $9x^2 + y^2 - z^2 - 2y + 2z = 0$

10.6 ANSWERS

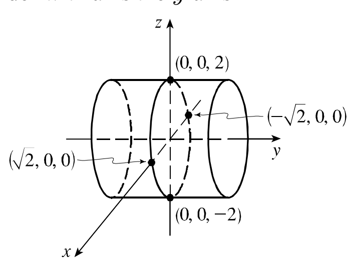
E Click here for exercises.

S Click here for solutions.

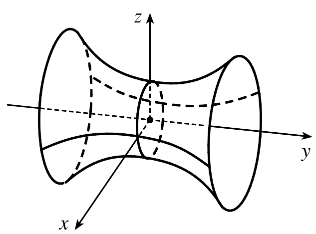
1. $x = k$, $y^2 + z^2 = k$, circle $(k > 0)$;
$y = k$, $x - k^2 = z^2$, parabola; $z = k$, $x - k^2 = y^2$, parabola
Circular paraboloid with axis the x-axis



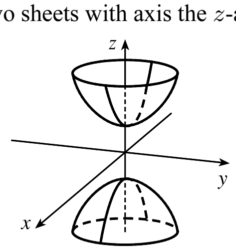
2. $x = k$, $z = \pm\sqrt{4 - 2k^2}$, two parallel lines $(|k| < \sqrt{2})$;
$y = k$, $2x^2 + z^2 = 4$, ellipse;
$z = k$, $x = \pm\sqrt{2 - (k^2/2)}$, two parallel lines $(|k| < 2)$
Elliptic cylinder with axis the y-axis



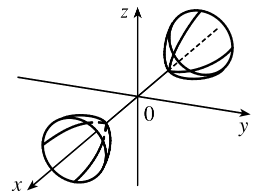
3. $x = k$, $z^2 - y^2 = 1 - k^2$, hyperbola;
$y = k$, $x^2 + z^2 = 1 + k^2$, circle;
$z = k$, $x^2 - y^2 = 1 - k^2$, hyperbola
Hyperboloid of one sheet with axis the y-axis



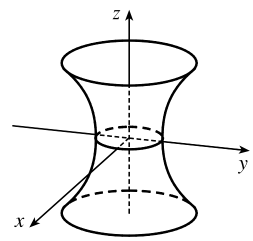
4. $x = k$, $4z^2 - y^2 = 1 + k^2$, hyperbola;
$y = k$, $4z^2 - x^2 = 1 + k^2$, hyperbola;
$z = k$, $x^2 + y^2 = 4k^2 - 1$, circle $(|k| > \frac{1}{2})$
Hyperboloid of two sheets with axis the z-axis



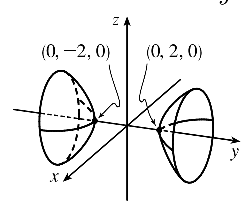
5. $x = k$, $y^2 + z^2 = 9\left(k^2 - 1\right)$, circle $(|k| > 1)$;
$y = k$, $9x^2 - z^2 = 9 + k^2$, hyperbola;
$z = k$, $9x^2 - y^2 = 9 + k^2$, hyperbola
Hyperboloid of two sheets with axis the x-axis



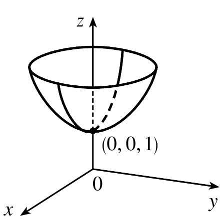
6. $\dfrac{x^2}{4} + \dfrac{y^2}{3} - \dfrac{z^2}{12} = 1$
Hyperboloid of one sheet with axis the z-axis



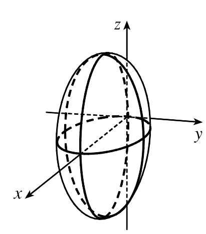
7. $-\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} - \dfrac{z^2}{6^2} = 1$
Hyperboloid of two sheets with axis the y-axis



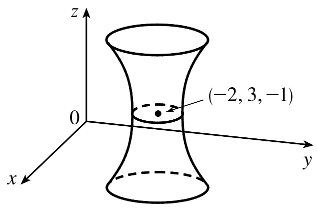
8. $z - 1 = x^2 + y^2$
Circular paraboloid with axis the z-axis



9. $(x - 1)^2 + 4y^2 + z^2 = 1$
Ellipsoid

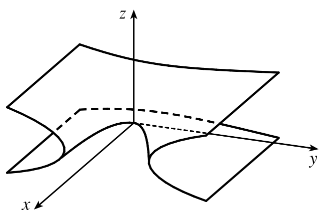


10. $(x+2)^2 + (y-3)^2 - 4(z+1)^2 = 22$
Hyperboloid of one sheet with center $(-2, 3, -1)$ and axis parallel to the z-axis

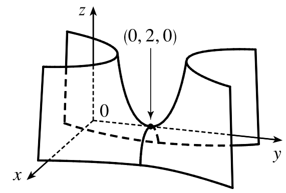


11. $x = \dfrac{y^2}{2^2} - \dfrac{z^2}{(\sqrt{2})^2}$

Hyperbolic paraboloid with saddle point $(0, 0, 0)$

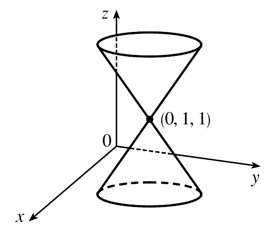


12. $z = (y-2)^2 - x^2$
Hyperbolic paraboloid with center $(0, 2, 0)$



13. $(z-1)^2 = \dfrac{x^2}{(1/3)^2} + (y-1)^2$

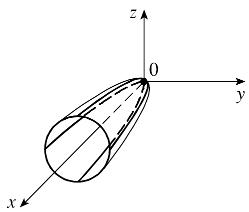
Elliptic cone with axis parallel to the z-axis



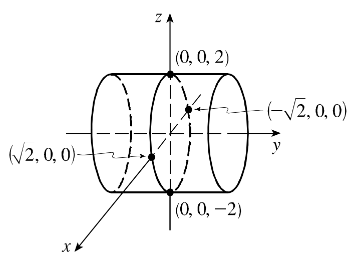
10.6 SOLUTIONS

E Click here for exercises.

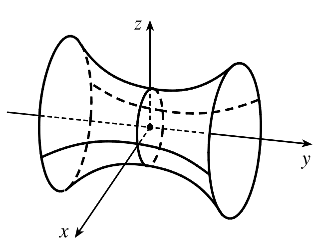
1. Traces: $x = k$, $y^2 + z^2 = k$, a circle for $k > 0$; $y = k$, $x - k^2 = z^2$, a parabola; $z = k$, $x - k^2 = y^2$, a parabola. Thus the surface is a circular paraboloid (a paraboloid of revolution) with axis the x-axis and vertex at $(0, 0, 0)$.



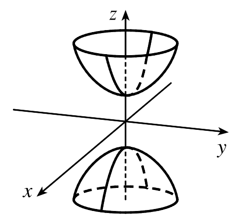
2. Traces: $x = k$, $z^2 = 4 - 2k^2$ or $z = \pm\sqrt{4 - 2k^2}$, two parallel lines for $|k| < \sqrt{2}$; $y = k$, $2x^2 + z^2 = 4$, an ellipse; $z = k$, $2x^2 = 4 - k^2$ or $x = \pm\sqrt{2 - (k^2/2)}$, two parallel lines for $|k| < 2$. Thus the surface is an elliptic cylinder with axis the y-axis.



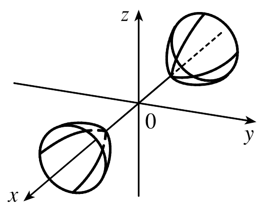
3. The trace in any plane $x = k$ is given by $z^2 - y^2 = 1 - k^2$, $x = k$ whose graph is a hyperbola. The trace in any plane $y = k$ is the circle given by $x^2 + z^2 = 1 + k^2$, $y = k$, and the trace in any plane $z = k$ is the hyperbola given by $x^2 - y^2 = 1 - k^2$, $z = k$. Thus the surface is a hyperboloid of one sheet with axis the y-axis.



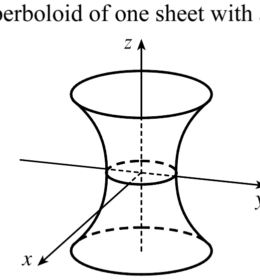
4. Traces: $x = k$, $4z^2 - y^2 = 1 + k^2$, a hyperbola; $y = k$, $4z^2 - x^2 = 1 + k^2$, a hyperbola; $z = k$, $-x^2 - y^2 = 1 - 4k^2$ or $x^2 + y^2 = 4k^2 - 1$, a circle for $k > \frac{1}{2}$ or $k < -\frac{1}{2}$. Thus the surface is a hyperboloid of two sheets with axis the z-axis.



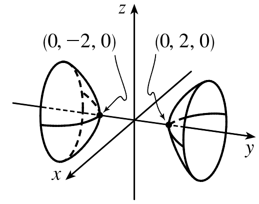
5. Traces: $x = k$, $y^2 + z^2 = 9\left(k^2 - 1\right)$, a circle for $|k| > 1$; $y = k$, $9x^2 - z^2 = 9 + k^2$, a hyperbola; $z = k$, $9x^2 - y^2 = 9 + k^2$, a hyperbola. Thus the surface is a hyperboloid of two sheets with axis the x-axis.



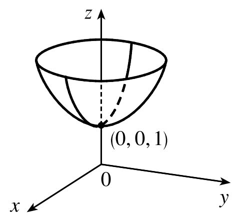
6. $z^2 = 3x^2 + 4y^2 - 12$ or $3x^2 + 4y^2 - z^2 = 12$ or $\dfrac{x^2}{4} + \dfrac{y^2}{3} - \dfrac{z^2}{12} = 1$ or $\dfrac{x^2}{2^2} + \dfrac{y^2}{\left(\sqrt{3}\right)^2} - \dfrac{z^2}{\left(\sqrt{12}\right)^2} = 1$ represents a hyperboloid of one sheet with axis the z-axis.



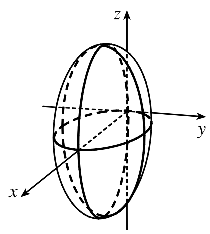
7. $4x^2 - 9y^2 + z^2 + 36 = 0$ or $-4x^2 + 9y^2 - z^2 = 36$ or $-\dfrac{x^2}{3^2} + \dfrac{y^2}{2^2} - \dfrac{z^2}{6^2} = 1$, a hyperboloid of two sheets with axis the y-axis.



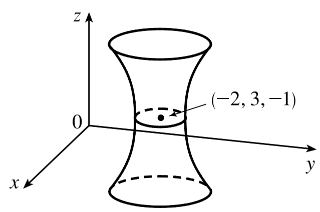
8. $z = x^2 + y^2 + 1$ or $z - 1 = x^2 + y^2$, a circular paraboloid with axis the z-axis and vertex $(0, 0, 1)$.



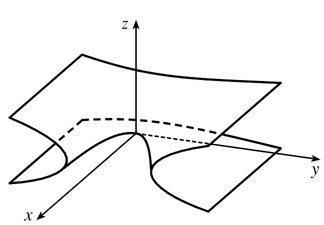
9. Completing the square in x gives $(x-1)^2 + 4y^2 + z^2 = 1$ or $(x-1)^2 + \dfrac{y^2}{(1/2)^2} + z^2 = 1$, an ellipsoid with center $(1, 0, 0)$ and intercepts $(0, 0, 0)$, $(2, 0, 0)$.



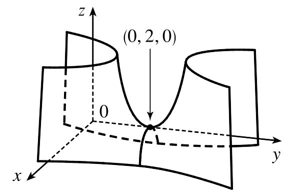
10. Completing the square in all three variables gives $(x+2)^2 + (y-3)^2 - 4(z+1)^2 = 13 + 9$ or $\dfrac{(x+2)^2}{(\sqrt{22})^2} + \dfrac{(y-3)^2}{(\sqrt{22})^2} - \dfrac{(z+1)^2}{(\frac{1}{2}\sqrt{22})^2} = 1$, a hyperboloid of one sheet with center $(-2, 3, -1)$ and axis the vertical line $y = 3$, $x = -2$.



11. $4x = y^2 - 2z^2$ or $x = \dfrac{y^2}{2^2} - \dfrac{z^2}{(\sqrt{2})^2}$, a hyperbolic paraboloid with saddle point $(0, 0, 0)$. The traces in the xy-, yz-, and xz-planes are respectively $x = \dfrac{y^2}{2^2}$ (a parabola), $\dfrac{y^2}{2^2} = \dfrac{z^2}{(\sqrt{2})^2}$ (two intersecting lines), and $x = -\dfrac{z^2}{(\sqrt{2})^2}$ (a parabola).



12. Completing the square in y gives $x^2 - (y-2)^2 + z = 4 - 4 = 0$ or $z = (y-2)^2 - x^2$, a hyperbolic paraboloid with center at $(0, 2, 0)$.



13. Completing the squares in y and z gives $9x^2 + (y-1)^2 - (z-1)^2 = 1 - 1 = 0$ or $(z-1)^2 = \dfrac{x^2}{(1/3)^2} + (y-1)^2$, an elliptic cone with axis parallel to the z-axis and vertex $(0, 1, 1)$.