11.5 THE CHAIN RULE

A Click here for answers.

S Click here for solutions.

1–8 ▪ Use the Chain Rule to find dz/dt or dw/dt.

1. $z = x^2 + y^2, \quad x = t^3, \quad y = 1 + t^2$

2. $z = x^2y^3, \quad x = 1 + \sqrt{t}, \quad y = 1 - \sqrt{t}$

3. $z = \ln(x + y^2), \quad x = \sqrt{1 + t}, \quad y = 1 + \sqrt{t}$

4. $z = xe^{x/y}, \quad x = \cos t, \quad y = e^{2t}$

5. $z = 6x^3 - 3xy + 2y^2, \quad x = e^t, \quad y = \cos t$

6. $z = x\sqrt{1 + y^2}, \quad x = te^{2t}, \quad y = e^{-t}$

7. $w = xy^2z^3, \quad x = \sin t, \quad y = \cos t, \quad z = 1 + e^{2t}$

8. $w = \dfrac{x}{y} + \dfrac{y}{z}, \quad x = \sqrt{t}, \quad y = \cos 2t, \quad z = e^{-3t}$

9–14 ▪ Use the Chain Rule to find $\partial z/\partial s$ and $\partial z/\partial t$.

9. $z = x^2 \sin y, \quad x = s^2 + t^2, \quad y = 2st$

10. $z = \sin x \cos y, \quad x = (s - t)^2, \quad y = s^2 - t^2$

11. $z = x^2 - 3x^2y^3, \quad x = se^t, \quad y = se^{-t}$

12. $z = x \tan^{-1}(xy), \quad x = t^2, \quad y = se^t$

13. $z = 2^{x-3y}, \quad x = s^2t, \quad y = st^2$

14. $z = xe^y + ye^{-x}, \quad x = e^t, \quad y = st^2$

15–22 ▪ Use the Chain Rule to find the indicated partial derivatives.

15. $w = x^2 + y^2 + z^2, \; x = st, \; y = s\cos t, \; z = s\sin t;$
$\dfrac{\partial w}{\partial s}, \dfrac{\partial w}{\partial t}$ when $s = 1, t = 0$

16. $u = xy + yz + zx, \; x = st, \; y = e^{st}, \; z = t^2;$
$\dfrac{\partial u}{\partial s}, \dfrac{\partial u}{\partial t}$ when $s = 0, t = 1$

17. $z = y^2 \tan x, \; x = t^2uv, \; y = u + tv^2;$
$\dfrac{\partial z}{\partial t}, \dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}$ when $t = 2, u = 1, v = 0$

18. $z = \dfrac{x}{y}, \; x = re^{st}, \; y = rse^t;$
$\dfrac{\partial z}{\partial r}, \dfrac{\partial z}{\partial s}, \dfrac{\partial z}{\partial t}$ when $r = 1, s = 2, t = 0$

19. $u = \dfrac{x + y}{y + z}, \; x = p + r + t, \; y = p - r + t, \; z = p + r - t;$
$\dfrac{\partial u}{\partial p}, \dfrac{\partial u}{\partial r}, \dfrac{\partial u}{\partial t}$

20. $t = z\sec(xy), \; x = uv, \; y = vw, \; z = wu; \quad \dfrac{\partial t}{\partial u}, \dfrac{\partial t}{\partial v}, \dfrac{\partial t}{\partial w}$

21. $w = \cos(x - y), \; x = rs^2t^3 \sin\theta, \; y = r^2st\cos\theta;$
$\dfrac{\partial w}{\partial r}, \dfrac{\partial w}{\partial s}, \dfrac{\partial w}{\partial t}, \dfrac{\partial w}{\partial \theta}$

22. $u = pq - p^2r^2s, \; p = x + 2y, \; q = x - 2y, \; r = \dfrac{x}{y^4},$
$s = 2xy^{3/2}; \quad \dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}$

23–26 ▪ Use Equation 6 to find dy/dx.

23. $x^2 - xy + y^3 = 8$

24. $y^5 + 3x^2y^2 + 5x^4 = 12$

25. $x\cos y + y\cos x = 1$

26. $2y^2 + \sqrt[3]{xy} = 3x^2 + 17$

27–33 ▪ Use Equations 7 to find $\partial z/\partial x$ and $\partial z/\partial y$.

27. $xy + yz - xz = 0$

28. $x^2 + y^2 - z^2 = 2x(y + z)$

29. $xy^2z^3 + x^3y^2z = x + y + z$

30. $y^2ze^{x+y} - \sin(xyz) = 0$

31. $xy^2 + yz^2 + zx^2 = 3$

32. $xe^y + yz + ze^x = 0$

33. $\ln(x + yz) = 1 + xy^2z^3$

34. The radius of a right cylinder is decreasing at a rate of 1.2 cm/s while its height is increasing at a rate of 3 cm/s. At what rate is the volume of the cylinder changing when the radius is 80 cm and the height is 150 cm?

11.5 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. $6t^5 + 4t^3 + 4t$

2. $\dfrac{(1-t)\left(1-\sqrt{t}\right)^2}{\sqrt{t}} - \dfrac{3(1-t)^2}{2\sqrt{t}}$

3. $\dfrac{1}{\sqrt{1+t}+1+\sqrt{t}}\left(\dfrac{1}{2\sqrt{1+t}} + \dfrac{1+\sqrt{t}}{\sqrt{t}}\right)$

4. $-e^{\cos t/e^{2t}}\left[\left(1+\dfrac{\cos t}{e^{2t}}\right)\sin t - \dfrac{2e^{2t}\cos^2 t}{e^{4t}}\right]$

5. $\left(18e^{2t} - 3\cos t\right)e^t + \left(3e^t - 4\cos t\right)\sin t$

6. $e^{2t}\sqrt{1+e^{-2t}}\,(1+2t) - t\sqrt{1+e^{-2t}}$

7. $y^2z^3(\cos t) + 2xyz^3(-\sin t) + 3xy^2z^2\left(2e^{2t}\right)$

8. $\dfrac{1}{2y\sqrt{t}} + 2(\sin 2t)\left(\dfrac{x}{y^2} - \dfrac{1}{z}\right) + \dfrac{3y}{z^2e^{3t}}$

9. $4sx\sin y + 2tx^2\cos y,\ 4xt\sin y + 2sx^2\cos y$

10. $2(s-t)\cos x\cos y - 2s\sin x\sin y,$
$2(t-s)\cos x\cos y + 2t\sin x\sin y$

11. $\left(2x - 6xy^3\right)e^t - 9x^2y^2e^{-t},\ \left(2x-6xy^3\right)se^t + 9x^2y^2se^{-t}$

12. $\dfrac{x^2e^t}{1+x^2y^2},\ \left[\tan^{-1}(xy) + \dfrac{xy}{1+x^2y^2}\right](2t) + \dfrac{x^2}{1+x^2y^2}se^t$

13. $\left(2^{x-3y}\ln 2\right)\left(2st - 3t^2\right),\ \left(2^{x-3y}\ln 2\right)\left(s^2 - 6st\right)$

14. $\left(xe^y + e^{-x}\right)t^2,\ \left(e^y - ye^{-x}\right)e^t + 2\left(xe^y + e^{-x}\right)st$

15. 2, 0

16. 3, 2

17. 0, 0, 4

18. $0, -\frac{1}{4}, \frac{1}{2}$

19. $-t/\left(p^2\right), 0, 1/p$

20. $\sec(xy)\,[w + vzy\tan(xy)],\ z\sec(xy)\tan(xy)\,[yu + xw],$
$\sec(xy)\,[u + vzx\tan(xy)]$

21. $st\sin(x-y)\left[2r\cos\theta - st^2\sin\theta\right],$
$[rt\sin(x-y)]\left(r\cos\theta - 2st^2\sin\theta\right),$
$[sr\sin(x-y)]\left(r\cos\theta - 3st^2\sin\theta\right),$
$[-rst\sin(x-y)]\left(st^2\cos\theta + r\sin\theta\right)$

22. $2x - \dfrac{8x^2(x+2y)(x+y)}{y^{13/2}},$
$-8y + \dfrac{(x+2y)\,x^3y^{1/2}(5x+2y)}{y^8}$

23. $\dfrac{y-2x}{3y^2-x}$

24. $-\dfrac{6xy^2+20x^3}{5y^4+6x^2y}$

25. $\dfrac{y\sin x - \cos y}{\cos x - x\sin y}$

26. $\dfrac{18x - x^{-2/3}y^{1/3}}{12y + x^{1/3}y^{-2/3}}$

27. $\dfrac{z-y}{y-x},\ \dfrac{x+z}{x-y}$

28. $\dfrac{x-y-z}{z+x},\ \dfrac{y-x}{z+x}$

29. $-\dfrac{y^2z^3 + 3x^2y^2z - 1}{3xy^2z^2 + x^3y^2 - 1},\ -\dfrac{2xyz^3 + 2x^3yz - 1}{3xy^2z^2 + x^3y^2 - 1}$

30. $\dfrac{z\cos(xyz) - yze^{x+y}}{ye^{x+y} - x\cos(xyz)},\ \dfrac{xz\cos(xyz) - e^{x+y}\left(2yz + y^2z\right)}{y^2e^{x+y} - xy\cos(xyz)}$

31. $-\dfrac{y^2+2zx}{2yz+x^2},\ -\dfrac{2xy+z^2}{2yz+x^2}$

32. $-\dfrac{e^y + ze^x}{y+e^x},\ -\dfrac{xe^y+z}{y+e^x}$

33. $\dfrac{y^2z^3(x+yz) - 1}{y - 3xy^2z^2(x+yz)},\ \dfrac{2xyz^3(x+yz) - z}{y - 3xy^2z^2(x+yz)}$

34. $-9600\pi\ \text{cm}^3/\text{s}$

11.5 SOLUTIONS

E Click here for exercises.

1. $z = x^2 + y^2,\ x = t^3,\ y = 1 + t^2 \quad \Rightarrow$
$$\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = (2t^3)(3t^2) + 2(1+t^2)(2t) = 6t^5 + 4t^3 + 4t$$

2. $z = x^2y^3,\ x = 1 + \sqrt{t},\ y = 1 - \sqrt{t} \quad \Rightarrow$
$$\frac{dz}{dt} = 2xy^3\frac{dx}{dt} + 3x^2y^2\frac{dy}{dt} = 2xy^3\frac{1}{2\sqrt{t}} + 3x^2y^2\left(-\frac{1}{2\sqrt{t}}\right) = 2(1+\sqrt{t})(1-\sqrt{t})^3\frac{1}{2\sqrt{t}} + 3(1+\sqrt{t})^2(1-\sqrt{t})^2\left(-\frac{1}{2\sqrt{t}}\right) = \frac{(1-t)(1-\sqrt{t})^2}{\sqrt{t}} - \frac{3(1-t)^2}{2\sqrt{t}}$$

3. $z = \ln(x + y^2),\ x = \sqrt{1+t},\ y = 1 + \sqrt{t} \quad \Rightarrow$
$$\frac{dz}{dt} = \frac{1}{(x+y^2)}\frac{1}{2\sqrt{1+t}} + \frac{1}{(x+y^2)}2y\frac{1}{2\sqrt{t}} = \frac{1}{\sqrt{1+t}+1+\sqrt{t}}\left(\frac{1}{2\sqrt{1+t}} + \frac{1+\sqrt{t}}{\sqrt{t}}\right)$$

4. $z = xe^{x/y},\ x = \cos t,\ y = e^{2t} \quad \Rightarrow$
$$\frac{dz}{dt} = e^{x/y}\left(1 + \frac{x}{y}\right)(-\sin t) - x^2y^{-2}e^{x/y}(2e^{2t}) = -e^{\cos t/e^{2t}}\left[\left(1 + \frac{\cos t}{e^{2t}}\right)\sin t - \frac{2e^{2t}\cos^2 t}{e^{4t}}\right]$$

5. $z = 6x^3 - 3xy + 2y^2,\ x = e^t,\ y = \cos t \quad \Rightarrow$
$$\frac{dz}{dt} = (18x^2 - 3y)e^t + (-3x + 4y)(-\sin t) = (18e^{2t} - 3\cos t)e^t + (3e^t - 4\cos t)\sin t$$

6. $z = x\sqrt{1-y^2},\ x = te^{2t},\ y = e^{-t} \quad \Rightarrow$
$$\frac{\partial z}{\partial t} = \sqrt{1+y^2}(e^{2t} + 2te^{2t}) + \tfrac{1}{2}x(1+y^2)^{-1/2}(2y)(-e^{-t}) = e^{2t}\sqrt{1+e^{-2t}}(1+2t) - t\sqrt{1+e^{-2t}}$$

7. $w = xy^2z^3,\ x = \sin t,\ y = \cos t,\ z = 1 + e^{2t} \quad \Rightarrow$
$$\frac{dw}{dt} = y^2z^3(\cos t) + 2xyz^3(-\sin t) + 3xy^2z^2(2e^{2t})$$

8. $w = \dfrac{x}{y} + \dfrac{y}{z},\ x = \sqrt{t},\ y = \cos 2t,\ z = e^{-3t} \quad \Rightarrow$
$$\frac{dw}{dt} = \frac{1}{y}\frac{1}{2\sqrt{t}} + \left(\frac{-x}{y^2} + \frac{1}{z}\right)(-2\sin 2t) + \frac{-y}{z^2}(-3e^{-3t}) = \frac{1}{2y\sqrt{t}} + 2(\sin 2t)\left(\frac{x}{y^2} - \frac{1}{z}\right) + \frac{3y}{z^2e^{3t}}$$

9. $z = x^2\sin y,\ x = s^2 + t^2,\ y = 2st \quad \Rightarrow$
$$\frac{\partial z}{\partial s} = (2x\sin y)(2s) + (x^2\cos y)(2t) = 4sx\sin y + 2tx^2\cos y$$
$$\frac{\partial z}{\partial t} = (2x\sin y)(2t) + (x^2\cos y)(2s) = 4xt\sin y + 2sx^2\cos y$$

10. $z = \sin x\cos y,\ x = (s-t)^2,\ y = s^2 - t^2 \quad \Rightarrow$
$$\frac{\partial z}{\partial s} = (\cos x\cos y)\,2(s-t) - (\sin x\sin y)(2s) = 2(s-t)\cos x\cos y - (2s)\sin x\sin y$$
$$\frac{\partial z}{\partial t} = (\cos x\cos y)(-2)(s-t) - (\sin x\sin y)(-2t) = 2(t-s)\cos x\cos y + 2t\sin x\sin y$$

11. $z = x^2 - 3x^2y^3,\ x = se^t,\ y = se^{-t} \quad \Rightarrow$
$$\partial z/\partial s = (2x - 6xy^3)(e^t) + (-9x^2y^2)(e^{-t}) = (2x - 6xy^3)e^t - 9x^2y^2e^{-t}$$
$$\partial z/\partial t = (2x - 6xy^3)(se^t) + (-9x^2y^2)(-se^{-t}) = (2x - 6xy^3)se^t + 9x^2y^2se^{-t}$$

12. $z = x\tan^{-1}(xy),\ x = t^2,\ y = se^t \quad \Rightarrow$
$$\frac{\partial z}{\partial s} = \left[\tan^{-1}(xy) + \frac{x}{1+x^2y^2}y\right](0) + \frac{x^2}{1+x^2y^2}e^t = \frac{x^2e^t}{1+x^2y^2}$$
$$\frac{\partial z}{\partial t} = \left[\tan^{-1}(xy) + \frac{xy}{1+x^2y^2}\right](2t) + \frac{x^2}{1+x^2y^2}se^t$$

13. $z = 2^{x-3y},\ x = s^2t,\ y = st^2 \quad \Rightarrow$
$$\partial z/\partial s = (z\ln 2)(2st) + z(-3\ln 2)(t^2) = (2^{x-3y}\ln 2)(2st - 3t^2)$$
$$\partial z/\partial t = (z\ln 2)(s^2) + z(-3\ln 2)(2st) = (2^{x-3y}\ln 2)(s^2 - 6st)$$

14. $z = xe^y + ye^{-x},\ x = e^t,\ y = st^2 \quad \Rightarrow$
$$\partial z/\partial s = (e^y - ye^{-x})(0) + (xe^y + e^{-x})(t^2) = (xe^y + e^{-x})t^2$$
$$\partial z/\partial t = (e^y - ye^{-x})(e^t) + (xe^y + e^{-x})(2st)$$

15. $w = x^2 + y^2 + z^2,\ x = st,\ y = s\cos t,\ z = s\sin t \quad \Rightarrow$
$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial s} = 2xt + 2y\cos t + 2z\sin t$$
When $s = 1$, $t = 0$, we have $x = 0$, $y = 1$ and $z = 0$, so $\partial w/\partial s = 2\cos 0 = 2$. Similarly
$$\partial w/\partial t = 2xs + 2y(-s\sin t) + 2z(s\cos t) = 0 + (-2)\sin 0 + 0 = 0$$
when $s = 1$ and $t = 0$.

16. $u = xy + yz + zx,\ x = st,\ y = e^{st},\ z = t^2 \quad \Rightarrow$
$\partial u/\partial s = (y+z)\,t + (x+z)\,te^{st} + (x+y)\,(0)$ and
$\partial u/\partial t = (y+z)\,s + (x+z)\,se^{st} + (x+y)\,(2t)$. When
$s = 0,\ t = 1$, we have $x = 0,\ y = 1,\ z = 1$, so
$\partial u/\partial s = 2 + 1 + 0 = 3$ and $\partial u/\partial t = 0 + 0 + (1)\,(2) = 2$.

17. $z = y^2 \tan x,\ x = t^2 uv,\ y = u + tv^2 \quad \Rightarrow$
$\partial z/\partial t = \left(y^2 \sec^2 x\right) 2tuv + (2y \tan x)\, v^2$,
$\partial z/\partial u = \left(y^2 \sec^2 x\right) t^2 v + 2y \tan x$,
$\partial z/\partial v = \left(y^2 \sec^2 x\right) t^2 u + (2y \tan x)\, 2tv$. When $t = 2$,
$u = 1$ and $v = 0$, we have $x = 0,\ y = 1$, so $\partial z/\partial t = 0$,
$\partial z/\partial u = 0,\ \partial z/\partial v = 4$.

18. $z = \dfrac{x}{y},\ x = re^{st},\ y = rse^t \quad \Rightarrow$
$\dfrac{\partial z}{\partial r} = \dfrac{1}{y} e^{st} + \dfrac{-x}{y^2} se^t,\ \dfrac{\partial z}{\partial s} = \dfrac{1}{y} rte^{st} - \dfrac{x}{y^2} re^t,$
$\dfrac{\partial z}{\partial t} = \dfrac{1}{y} rse^{st} - \dfrac{x}{y^2} rse^t$. When $r = 1,\ s = 2$ and $t = 0$, we
have $x = 1,\ y = 2$, so $\partial z/\partial r = \frac{1}{2} + \frac{-1}{4} \cdot 2 = 0$,
$\partial z/\partial s = 0 - \frac{1}{4} = -\frac{1}{4}$ and $\partial z/\partial t = \frac{1}{2} \cdot 2 - \frac{1}{4} \cdot 2 = \frac{1}{2}$.

19. $u = \dfrac{x+y}{y+z},\ x = p + r + t,\ y = p - r + t,\ z = p + r - t \Rightarrow$

$$\frac{\partial u}{\partial p} = \frac{1}{y+z} + \frac{(y+z)-(x+y)}{(y+z)^2} - \frac{x+y}{(y+z)^2} = \frac{(y+z)+(z-x)-(x+y)}{(y+z)^2} = 2\frac{z-x}{(y+z)^2} = 2\frac{-2t}{4p^2} = -\frac{t}{p^2}$$

$$\frac{\partial u}{\partial r} = \frac{1}{y+z} + \frac{z-x}{(y+z)^2}(-1) - \frac{x+y}{(y+z)^2} = 0, \text{ and}$$

$$\frac{\partial u}{\partial t} = \frac{1}{y+z} + \frac{z-x}{(y+z)^2} + \frac{x+y}{(y+z)^2} = 2\frac{y+z}{(y+z)^2} = \frac{2}{2p} = \frac{1}{p}$$

20. $t = z \sec(xy),\ x = uv,\ y = vw,\ z = wu \quad \Rightarrow$

$$\frac{\partial t}{\partial u} = [zy \sec(xy) \tan(xy)]\, v + [zx \sec(xy) \tan(xy)]\,(0) + [\sec(xy)]\, w = \sec(xy)\,[w + vzy \tan(xy)]$$

$$\frac{\partial t}{\partial v} = [zy \sec(xy) \tan(xy)]\, u + [zx \sec(xy) \tan(xy)]\, w + [\sec(xy)]\,(0) = z \sec(xy) \tan(xy)\,[yu + xw]$$

$$\frac{\partial t}{\partial w} = [zy \sec(xy) \tan(xy)]\,(0) + [zx \sec(xy) \tan(xy)]\, v + [\sec(xy)]\, u = \sec(xy)\,[u + vzx \tan(xy)]$$

21.

$$\frac{\partial w}{\partial r} = [-\sin(x-y)]\, s^2 t^3 \sin\theta + [\sin(x-y)]\, 2rst \cos\theta = st \sin(x-y)\left[2r\cos\theta - st^2 \sin\theta\right]$$

$$\frac{\partial w}{\partial s} = [-\sin(x-y)]\, 2rst^3 \sin\theta + [\sin(x-y)]\, r^2 t \cos\theta = [rt \sin(x-y)]\left(r\cos\theta - 2st^2 \sin\theta\right)$$

$$\frac{\partial w}{\partial t} = [-\sin(x-y)]\, 3rs^2 t^2 \sin\theta + [\sin(x-y)]\, r^2 s \cos\theta = [sr \sin(x-y)]\left(r\cos\theta - 3st^2 \sin\theta\right)$$

$$\frac{\partial w}{\partial \theta} = [-\sin(x-y)]\, rs^2 t^3 \cos\theta + [\sin(x-y)]\left(-r^2 st \sin\theta\right) = [-rst \sin(x-y)]\left(st^2 \cos\theta + r\sin\theta\right)$$

22.

$$\frac{\partial u}{\partial x} = \left(q - 2pr^2 s\right) + p + \left(-2p^2 rs\right)\frac{1}{y^4} + \left(-p^2 r^2\right)\left(2y^{3/2}\right) = p + q - 2pr^2 s - \frac{2p^2 rs}{y^4} - 2p^2 r^2 y^{3/2} = 2x - \frac{8x^2\,(x+2y)\,(x+y)}{y^{13/2}}$$

$$\frac{\partial u}{\partial y} = \left(q - 2pr^2 s\right)(2) + p\,(-2) + \left(-2p^2 rs\right)\left(-\frac{4x}{y^5}\right) + \left(-p^2 r^2\right) 3xy^{1/2} = 2\,(q-p) - 4pr^2 s + \frac{8p^2 rsx}{y^5} - 3p^2 r^2 xy^{1/2} = -8y + \frac{(x+2y)\,x^3 y^{1/2}\,(5x+2y)}{y^8}$$

23. $x^2 - xy + y^3 = 8$, so let $F(x,y) = x^2 - xy + y^3 - 8 = 0$.
Then $\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = \dfrac{-(2x-y)}{-x+3y^2} = \dfrac{y-2x}{3y^2-x}$.

24. $y^5 + 3x^2y^2 + 5x^4 = 12$, so let
$F(x,y) = y^5 + 3x^2y^2 + 5x^4 - 12 = 0$. Then
$\dfrac{dy}{dx} = -\dfrac{F_x}{F_y} = -\dfrac{6xy^2 + 20x^3}{5y^4 + 6x^2y}$.

25. $x\cos y + y\cos x = 1$, so let
$F(x,y) = x\cos y + y\cos x - 1 = 0$. Then
$\dfrac{dy}{dx} = -\dfrac{\cos y - y\sin x}{-x\sin y + \cos x} = \dfrac{y\sin x - \cos y}{\cos x - x\sin y}$.

26. $2y^2 + \sqrt[3]{xy} = 3x^2 + 17$, so let
$F(x,y) = 2y^2 + \sqrt[3]{xy} - 3x^2 - 17 = 0$. Then
$$\frac{dy}{dx} = -\frac{y\Big/\left[3\,(xy)^{2/3}\right] - 6x}{4y + x\Big/\left[3\,(xy)^{2/3}\right]} = \frac{18x - x^{-2/3}y^{1/3}}{12y + x^{1/3}y^{-2/3}}.$$

27. Let $F(x,y,z) = xy + yz - xz = 0$. Then
$\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z} = -\dfrac{y-z}{y-x} = \dfrac{z-y}{y-x}$,
$\dfrac{\partial z}{\partial y} = -\dfrac{F_y}{F_z} = -\dfrac{x+z}{y-x} = \dfrac{x+z}{x-y}$.

28. $x^2+y^2-z^2=2x(y+z)$. Let $F(x,y,z)=x^2+y^2-z^2-2x(y+z)=0$. Then
$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{2x-2y-2z}{-2z-2x}=\frac{x-y-z}{z+x},$$
$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{2y-2x}{-2z-2x}=\frac{y-x}{z+x}$$

29. $xy^2z^3+x^3y^2z=x+y+z$. Let $F(x,y,z)=xy^2z^3+x^3y^2z-(x+y+z)$.
Then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}=-\dfrac{y^2z^3+3x^2y^2z-1}{3xy^2z^2+x^3y^2-1}$,
$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{2xyz^3+2x^3yz-1}{3xy^2z^2+x^3y^2-1}.$$

30. Let $F(x,y,z)=y^2ze^{x+y}-\sin(xyz)=0$. Then
$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{y^2ze^{x+y}-yz\cos(xyz)}{y^2e^{x+y}-xy\cos(xyz)}=\frac{z\ \cos(xyz)-yze^{x+y}}{ye^{x+y}-x\cos(xyz)}$$
$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=\frac{xz\cos(xyz)-e^{x+y}\left(2yz+y^2z\right)}{y^2e^{x+y}-xy\cos(xyz)}$$

31. $xy^2+yz^2+zx^2=3$, so let $F(x,y)=xy^2+yz^2+zx^2-3=0$.
Then $\dfrac{\partial z}{\partial x}=-\dfrac{F_x}{F_z}=-\dfrac{y^2+2zx}{2yz+x^2}$ and
$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{2xy+z^2}{2yz+x^2}.$$

32. Let $F(x,y,z)=xe^y+yz+ze^x=0$. Then
$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{e^y+ze^x}{y+e^x},\ \frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{xe^y+z}{y+e^x}.$$

33. $\ln(x+yz)=1+xy^2z^3$, so let $F(x,y)=\ln(x+yz)-1-xy^2z^3=0$. Then
$$\frac{\partial z}{\partial x}=-\frac{F_x}{F_z}=-\frac{1/(x+yz)-y^2z^3}{y/(x+yz)-3xy^2z^2}=\frac{y^2z^3(x+yz)-1}{y-3xy^2z^2(x+yz)}$$
$$\frac{\partial z}{\partial y}=-\frac{F_y}{F_z}=-\frac{z/(x+yz)-2xyz^3}{y/(x+yz)-3xy^2z^2}=\frac{2xyz^3(x+yz)-z}{y-3xy^2z^2(x+yz)}$$

34. $dr/dt=-1.2$, $dh/dt=3$, $V=\pi r^2h$ and $dV/dt=2\pi rh\,(dr/dt)+\pi r^2\,(dh/dt)$.
Thus when $r=80$ and $h=150$,
$dV/dt=(-28{,}800)\,\pi+(19{,}200)\,\pi=-9600\pi\ \text{cm}^3/\text{s}$.