13.5 CURL AND DIVERGENCE

A Click here for answers.

S Click here for solutions.

1–15 ■ Find (a) the curl and (b) the divergence of the vector field.

1. $\mathbf{F}(x, y, z) = xy\,\mathbf{i} + yz\,\mathbf{j} + zx\,\mathbf{k}$

2. $\mathbf{F}(x, y, z) = (x - 2z)\,\mathbf{i} + (x + y + z)\,\mathbf{j} + (x - 2y)\,\mathbf{k}$

3. $\mathbf{F}(x, y, z) = xe^{y}\,\mathbf{j} + ye^{z}\,\mathbf{k}$

4. $\mathbf{F}(x, y, z) = \dfrac{x}{z}\,\mathbf{i} + \dfrac{y}{z}\,\mathbf{j} - \dfrac{1}{z}\,\mathbf{k}$

5. $\mathbf{F}(x, y, z) = xe^{yz}\,\mathbf{i} + ye^{xz}\,\mathbf{j} + ze^{xy}\,\mathbf{k}$

6. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$

7. $\mathbf{F}(x, y, z) = x^2y\,\mathbf{i} + yz^2\,\mathbf{j} + zx^2\,\mathbf{k}$

8. $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$

9. $\mathbf{F}(x, y, z) = y^2z\,\mathbf{i} - x^2yz\,\mathbf{k}$

10. $\mathbf{F}(x, y, z) = xy\,\mathbf{j} + xyz\,\mathbf{k}$

11. $\mathbf{F}(x, y, z) = \sin x\,\mathbf{i} + \cos x\,\mathbf{j} + z^2\,\mathbf{k}$

12. $\mathbf{F}(x, y, z) = e^{xz}\,\mathbf{i} - 2e^{yz}\,\mathbf{j} + 3xe^{y}\,\mathbf{k}$

13. $\mathbf{F}(x, y, z) = (x + 3y - 5z)\,\mathbf{i} + (z - 3y)\,\mathbf{j} + (5x + 6y - z)\,\mathbf{k}$

14. $\mathbf{F}(x, y, z) = xe^{y}\,\mathbf{i} - ze^{-y}\,\mathbf{j} + y\ln z\,\mathbf{k}$

15. $\mathbf{F}(x, y, z) = e^{xyz}\,\mathbf{i} + \sin(x - y)\,\mathbf{j} - \dfrac{xy}{z}\,\mathbf{k}$

16–27 ■ Determine whether or not the given vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

16. $\mathbf{F}(x, y, z) = y\,\mathbf{i} + x\,\mathbf{j} + \mathbf{k}$

17. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + x\,\mathbf{k}$

18. $\mathbf{F}(x, y, z) = yz\,\mathbf{i} - z^2\,\mathbf{j} + x^2\,\mathbf{k}$

19. $\mathbf{F}(x, y, z) = z\,\mathbf{i} + 2yz\,\mathbf{j} + (x + y^2)\,\mathbf{k}$

20. $\mathbf{F}(x, y, z) = \cos y\,\mathbf{i} + \sin x\,\mathbf{j} + \tan z\,\mathbf{k}$

21. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + e^{y}\sin z\,\mathbf{j} + e^{y}\cos z\,\mathbf{k}$

22. $\mathbf{F}(x, y, z) = yz\,\mathbf{i} + (y^2 + xz)\,\mathbf{j} + xy\,\mathbf{k}$

23. $\mathbf{F}(x, y, z) = zx\,\mathbf{i} + xy\,\mathbf{j} + yz\,\mathbf{k}$

24. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$

25. $\mathbf{F}(x, y, z) = xy^2z^3\,\mathbf{i} + 2x^2yz^3\,\mathbf{j} + 3x^2y^2z^2\,\mathbf{k}$

26. $\mathbf{F}(x, y, z) = e^{x}\,\mathbf{i} + e^{z}\,\mathbf{j} + e^{y}\,\mathbf{k}$

27. $\mathbf{F}(x, y, z) = yze^{xz}\,\mathbf{i} + e^{xz}\,\mathbf{j} + xye^{xz}\,\mathbf{k}$

13.5 ANSWERS

E Click here for exercises.

S Click here for solutions.

1. (a) $-y\,\mathbf{i} - z\,\mathbf{j} - x\,\mathbf{k}$ (b) $x + y + z$

2. (a) $-3\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$ (b) 2

3. (a) $e^z\,\mathbf{i} + e^y\,\mathbf{k}$ (b) $xe^y + ye^z$

4. (a) $\dfrac{y}{z^2}\,\mathbf{i} - \dfrac{x}{z^2}\,\mathbf{j}$ (b) $\dfrac{2z+1}{z^2}$

5. (a) $x\left(ze^{xy} - ye^{xz}\right)\mathbf{i} + y\left(xe^{yz} - ze^{xy}\right)\mathbf{j} + z\left(ye^{xz} - xe^{yz}\right)\mathbf{k}$

(b) $e^{yz} + e^{xz} + e^{xy}$

6. (a) $\mathbf{0}$ (b) 3

7. (a) $-\left(2yz\,\mathbf{i} + 2xz\,\mathbf{j} + x^2\,\mathbf{k}\right)$

(b) $2xy + z^2 + x^2$

8. (a) $\mathbf{0}$ (b) 0

9. (a) $-x^2z\,\mathbf{i} + \left(y^2 + 2xyz\right)\mathbf{j} - 2yz\,\mathbf{k}$

(b) $-x^2y$

10. (a) $xz\,\mathbf{i} - yz\,\mathbf{j} + y\,\mathbf{k}$ (b) $x\,(1+y)$

11. (a) $-\sin x\,\mathbf{k}$

(b) $\cos x + 2z$

12. (a) $\left(3xe^y + 2ye^{yz}\right)\mathbf{i} + \left(xe^{xz} - 3e^y\right)\mathbf{j}$

(b) $ze^{xz} - 2ze^{yz}$

13. (a) $5\,\mathbf{i} - 0\,\mathbf{j} - 3\,\mathbf{k}$ (b) -3

14. (a) $\left(e^{-y} + \ln z\right)\mathbf{i} - xe^y\,\mathbf{k}$

(b) $e^y + ze^{-y} + \dfrac{y}{z}$

15. (a) $-\dfrac{x}{z}\,\mathbf{i} + \left(xye^{xyz} + \dfrac{y}{z}\right)\mathbf{j} + \left[\cos(x-y) - xze^{xyz}\right]\mathbf{k}$

(b) $yze^{xyz} - \cos(x-y) + \dfrac{xy}{z^2}$

16. $f(x,y,z) = xy + z + K$

17. Not conservative

18. Not conservative

19. $f(x,y,z) = xz + y^2z + K$

20. Not conservative

21. $f(x,y,z) = \frac{1}{2}x^2 + e^y \sin z + K$

22. $f(x,y,z) = xyz + \frac{1}{3}y^3 + K$

23. Not conservative

24. $f(x,y,z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2 + K$

25. Not conservative

26. Not conservative

27. $f(x,y,z) = ye^{xz} + K$

13.5 SOLUTIONS

E Click here for exercises.

1. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & yz & zx \end{vmatrix}$

$$= \left[\frac{\partial}{\partial y}(zx) - \frac{\partial}{\partial z}(yz)\right]\mathbf{i} - \left[\frac{\partial}{\partial x}(zx) - \frac{\partial}{\partial z}(xy)\right]\mathbf{j} + \left[\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xy)\right]\mathbf{k}$$

$$= (0-y)\,\mathbf{i} - (z-0)\,\mathbf{j} + (0-x)\,\mathbf{k} = -y\,\mathbf{i} - z\,\mathbf{j} - x\,\mathbf{k}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial y}(yz) + \frac{\partial}{\partial z}(zx) = y + z + x = x + y + z$

2. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x-2z & x+y+z & x-2y \end{vmatrix}$

$$= (-2-1)\,\mathbf{i} - (1+2)\,\mathbf{j} + (1-0)\,\mathbf{k} = -3\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F}$

$$= \frac{\partial}{\partial x}(x-2z) + \frac{\partial}{\partial y}(x+y+z) + \frac{\partial}{\partial z}(x-2y) = 1 + 1 + 0 = 2$$

3. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & xe^y & ye^z \end{vmatrix}$

$$= (e^z - 0)\,\mathbf{i} - (0-0)\,\mathbf{j} + (e^y - 0)\,\mathbf{k} = e^z\,\mathbf{i} + e^y\,\mathbf{k}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}(xe^y) + \frac{\partial}{\partial z}(ye^z) = xe^y + ye^z$

4. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x/z & y/z & -1/z \end{vmatrix}$

$$= \left(0 + \frac{y}{z^2}\right)\mathbf{i} - \left(0 + \frac{x}{z^2}\right)\mathbf{j} + (0-0)\,\mathbf{k} = \frac{y}{z^2}\,\mathbf{i} - \frac{x}{z^2}\,\mathbf{j}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F}$

$$= \frac{\partial}{\partial x}\left(\frac{x}{z}\right) + \frac{\partial}{\partial y}\left(\frac{y}{z}\right) + \frac{\partial}{\partial z}\left(-\frac{1}{z}\right) = \frac{1}{z} + \frac{1}{z} + \frac{1}{z^2} = \frac{2z+1}{z^2}$$

5. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xe^{yz} & ye^{xz} & ze^{xy} \end{vmatrix}$

$$= (xze^{xy} - xye^{xz})\,\mathbf{i} - (yze^{xy} - xye^{yz})\,\mathbf{j} + (yze^{xz} - xze^{yz})\,\mathbf{k}$$

$$= x\,(ze^{xy} - ye^{xz})\,\mathbf{i} + y\,(xe^{yz} - ze^{xy})\,\mathbf{j} + z\,(ye^{xz} - xe^{yz})\,\mathbf{k}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F}$

$$= \frac{\partial}{\partial x}(xe^{yz}) + \frac{\partial}{\partial y}(ye^{xz}) + \frac{\partial}{\partial z}(ze^{xy}) = e^{yz} + e^{xz} + e^{xy}$$

6. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix}$

$$= (0-0)\,\mathbf{i} + (0-0)\,\mathbf{j} + (0-0)\,\mathbf{k} = \mathbf{0}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + \frac{\partial}{\partial z}(z) = 1 + 1 + 1 = 3$

7. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2y & yz^2 & zx^2 \end{vmatrix}$

$$= (0 - 2yz)\,\mathbf{i} + (0 - 2xz)\,\mathbf{j} + \left(0 - x^2\right)\mathbf{k} = -\left(2yz\,\mathbf{i} + 2xz\,\mathbf{j} + x^2\,\mathbf{k}\right)$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial}{\partial x}\left(x^2y\right) + \frac{\partial}{\partial y}\left(yz^2\right) + \frac{\partial}{\partial z}\left(zx^2\right) = 2xy + z^2 + x^2$

8. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & xz & xy \end{vmatrix}$

$$= (x-x)\,\mathbf{i} + (y-y)\,\mathbf{j} + (z-z)\,\mathbf{k} = \mathbf{0}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy) = 0 + 0 + 0 = 0$

9. (a) $\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y^2z & 0 & -x^2yz \end{vmatrix}$

$$= \left(-x^2z - 0\right)\mathbf{i} + \left(y^2 + 2xyz\right)\mathbf{j} + (0 - 2yz)\,\mathbf{k} = -x^2z\,\mathbf{i} + \left(y^2 + 2xyz\right)\mathbf{j} - 2yz\,\mathbf{k}$$

(b) $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F} = \frac{\partial}{\partial x}\left(y^2z\right) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}\left(-x^2yz\right) = 0 + 0 - x^2y = -x^2y$

10. (a) $\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & xy & xyz \end{vmatrix}$

$= xz\,\mathbf{i} - yz\,\mathbf{j} + y\,\mathbf{k}$

(b) $\operatorname{div}\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial}{\partial x}(0) + \frac{\partial}{\partial y}(xy) + \frac{\partial}{\partial z}(xyz)$

$= 0 + x + xy = x(1+y)$

11. (a) $\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin x & \cos x & z^2 \end{vmatrix}$

$= (-\sin x + 0)\,\mathbf{k} = -\sin x\,\mathbf{k}$

(b) $\operatorname{div}\mathbf{F} = \nabla \cdot \mathbf{F}$

$= \frac{\partial}{\partial x}(\sin x) + \frac{\partial}{\partial y}(\cos x) + \frac{\partial}{\partial z}(z^2)$

$= \cos x + 2z$

12. (a) $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xz} & -2e^{yz} & 3xe^{y} \end{vmatrix}$

$= (3xe^{y} + 2ye^{yz})\,\mathbf{i} + (xe^{xz} - 3e^{y})\,\mathbf{j}$

(b) $\nabla \times \mathbf{F} = \frac{\partial}{\partial x}(e^{xz}) + \frac{\partial}{\partial y}(-2e^{yz}) + \frac{\partial}{\partial z}(3xe^{y})$

$= ze^{xz} - 2ze^{yz}$

13. (a) $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x+3y-5z & z-3y & 5x+6y-z \end{vmatrix}$

$= (6-1)\,\mathbf{i} + (-5-5)\,\mathbf{j} + (0-3)\,\mathbf{k}$

$= 5\,\mathbf{i} - 0\,\mathbf{j} - 3\,\mathbf{k}$

(b) $\nabla \times \mathbf{F} = \frac{\partial}{\partial x}(x+3y-5z) + \frac{\partial}{\partial y}(z-3y) + \frac{\partial}{\partial z}(5x+6y-z)$

$= 1 - 3 - 1 = -3$

14. (a) $\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xe^{y} & -ze^{-y} & y\ln z \end{vmatrix}$

$= \left(e^{-y} + \ln z\right)\mathbf{i} - xe^{y}\,\mathbf{k}$

(b) $\operatorname{div}\mathbf{F} = \frac{\partial}{\partial x}(xe^{y}) + \frac{\partial}{\partial y}\left(-ze^{-y}\right) + \frac{\partial}{\partial z}(y\ln z)$

$= e^{y} + ze^{-y} + \frac{y}{z}$

15. (a) $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xyz} & \sin(x-y) & -xy/z \end{vmatrix}$

$= -\frac{x}{z}\,\mathbf{i} + \left(xye^{xyz} + \frac{y}{z}\right)\mathbf{j}$

$+ \left(\cos(x-y) - xze^{xyz}\right)\mathbf{k}$

(b) $\nabla \times \mathbf{F} = \frac{\partial}{\partial x}(e^{xyz}) + \frac{\partial}{\partial y}\sin(x-y) + \frac{\partial}{\partial z}\left(-\frac{xy}{z}\right)$

$= yze^{xyz} - \cos(x-y) + \frac{xy}{z^2}$

16. $\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & x & 1 \end{vmatrix} = \mathbf{0}$ and $\mathbf{F}$ is defined on all of $\mathbb{R}^3$ with component functions which have continuous partial derivatives, so by (4), $\mathbf{F}$ is conservative. Thus there exists f such that $\mathbf{F} = \nabla f$. Then $f_x(x,y,z) = y$ implies $f(x,y,z) = xy + g(y,z)$ and $f_y(x,y,z) = x + g_y(y,z)$. But $f_y(x,y,z) = x$, so $g(y,z) = h(z)$ and $f(x,y,z) = xy + h(z)$. Thus $f_z(x,y,z) = h'(z)$ but $f_z(x,y,z) = 1$ so $h(z) = z + K$. Hence a potential for $\mathbf{F}$ is $f(x,y,z) = xy + z + K$.

17. $\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & x \end{vmatrix} = -\mathbf{j} \neq \mathbf{0}$

Hence $\mathbf{F}$ isn't conservative.

18. $\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & -z^2 & x^2 \end{vmatrix}$

$= 2z\,\mathbf{i} + (y-2x)\,\mathbf{j} - z\,\mathbf{k} \neq \mathbf{0}$

Hence $\mathbf{F}$ isn't conservative.

19. Since $\operatorname{curl}\mathbf{F} = (2y-2y)\,\mathbf{i} + (1-1)\,\mathbf{j} + (0-0)\,\mathbf{k} = \mathbf{0}$, $\mathbf{F}$ is defined on all of $\mathbb{R}^3$, and the partial derivatives of the components of $\mathbf{F}$ are continuous, $\mathbf{F}$ is conservative. Thus there exists f such that $\nabla f = \mathbf{F}$. Then $f_x(x,y,z) = z$ implies $f(x,y,z) = xz + g(y,z)$ and $f_y(x,y,z) = g_y(y,z)$. But $f_y(x,y,z) = 2yz$, so $g(y,z) = y^2 z + h(z)$ and $f(x,y,z) = xz + y^2 z + h(z)$. Then $f_z(x,y,z) = x + y^2 + h'(z)$, but $f_z(x,y,z) = x + y^2$ so $h(z) = K$ and $f(x,y,z) = xz + y^2 z + K$.

20. $\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \cos y & \sin x & \tan z \end{vmatrix} = (\cos x - \sin y)\,\mathbf{k} \neq \mathbf{0}$.

Hence $\mathbf{F}$ isn't conservative.

21. Since

$$\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & e^y \sin z & e^y \cos z \end{vmatrix}$$
$$= (e^y \cos\, z - e^y \cos z)\,\mathbf{i} + (0-0)\,\mathbf{j} + (0-0)\,\mathbf{k} = \mathbf{0}$$

$\mathbf{F}$ is defined on $\mathbb{R}^3$, and since the partial derivatives of the components of $\mathbf{F}$ are continuous, $\mathbf{F}$ is conservative. Thus there exists f such that $\nabla f = \mathbf{F}$. Then $f_x(x,y,z) = x$ implies $f(x,y,z) = \frac{1}{2}x^2 + g(y,z)$ and $f_y(x,y,z) = g_y(y,z)$. But $f_y(x,y,z) = e^y \sin z$, so $g(y,z) = e^y \sin z + h(z)$ and $f(x,y,z) = \frac{1}{2}x^2 + e^y \sin z + h(z)$. Thus $f_z(x,y,z) = e^y \cos z + h'(z)$. But $f_z(x,y,z) = e^y \cos z$ implies $h(z) = K$ and $f(x,y,z) = \frac{1}{2}x^2 + e^y \sin z + K$ is a potential for $\mathbf{F}$.

22. Since

$$\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & y^2+xz & xy \end{vmatrix}$$
$$= (x-x)\,\mathbf{i} + (y-y)\,\mathbf{j} + (z-z)\,\mathbf{k} = \mathbf{0}$$

$\mathbf{F}$ is defined on $\mathbb{R}^3$, and since the partial derivatives of the components of $\mathbf{F}$ are continuous, $\mathbf{F}$ is conservative. Thus there exists f such that $\nabla f = \mathbf{F}$. Then $f_x(x,y,z) = yz$ implies $f(x,y,z) = xyz + g(y,z)$ and $f_y(x,y,z) = xz + g_y(y,z)$. But $f_y(x,y,z) = xz + y^2$ so $g(y,z) = \frac{1}{3}y^3 + h(z)$ and $f(x,y,z) = xyz + \frac{1}{3}y^3 + h(z)$. Then $f_z(x,y,z) = xy + h'(z)$. But $f_z(x,y,z) = xy$ so $h(z) = K$. Hence $f(x,y,z) = xyz + \frac{1}{3}y^3 + K$ is a potential for $\mathbf{F}$.

23. $\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ zx & xy & yz \end{vmatrix} = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k} \neq \mathbf{0}$, so $\mathbf{F}$ isn't conservative.

24. $\operatorname{curl}\mathbf{F} = \mathbf{0}$ by Problem 6(a), $\mathbf{F}$ is defined on all of $\mathbb{R}^3$, and the partial derivatives of the component functions are continuous, so $\mathbf{F}$ is conservative. Thus there exists a function f such that $\nabla f = \mathbf{F}$. Then $f_x(x,y,z) = x$ implies $f(x,y,z) = \frac{1}{2}x^2 + g(y,z)$ and $f_y(x,y,z) = g_y(y,z)$. But $f_y(x,y,z) = y$, so $g(y,z) = \frac{1}{2}y^2 + h(z)$ and $f(x,y,z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + h(z)$. Thus $f_z(x,y,z) = h'(z)$ but $f_z(x,y,z) = z$ so $h(z) = \frac{1}{2}z^2 + K$ and $f(x,y,z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2 + K$.

25. $\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy^2z^3 & 2x^2yz^3 & 3x^2y^2z^2 \end{vmatrix}$

$$= \left(6x^2yz^2 - 6x^2yz^2\right)\mathbf{i} - \left(6xy^2z^2 - 3xy^2z^2\right)\mathbf{j} + \left(4xyz^3 - 2xyz^3\right)\mathbf{k} \neq \mathbf{0}$$

so $\mathbf{F}$ isn't conservative.

26. $\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^x & e^z & e^y \end{vmatrix}$

$$= (e^y - e^z)\,\mathbf{i} - (0-0)\,\mathbf{j} + (0-0)\,\mathbf{k} \neq \mathbf{0}$$

so $\mathbf{F}$ isn't conservative.

27. $\operatorname{curl}\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yze^{xz} & e^{xz} & xye^{xz} \end{vmatrix}$

$$= (xe^{xz} - xe^{xz})\,\mathbf{i} - \left[(xyze^{xz} + ye^{xz}) - (xyze^{xz} + ye^{xz})\right]\mathbf{j} + (ze^{xz} - ze^{xz})\,\mathbf{k}$$
$$= \mathbf{0}$$

$\mathbf{F}$ is defined on all of $\mathbb{R}^3$, and the partial derivatives of the component functions are continuous, so $\mathbf{F}$ is conservative. Thus there exists a function f such that $\nabla f = \mathbf{F}$. Then $f_x(x,y,z) = yze^{xz}$ implies $f(x,y,z) = ye^{xz} + g(y,z)$ and $f_y(x,y,z) = e^{xz} + g_y(y,z)$. But $f_y(x,y,z) = e^{xz}$, so $g(y,z) = h(z)$ and $f(x,y,z) = ye^{xz} + h(z)$. Thus $f_z(x,y,z) = xye^{xz} + h'(z)$ but $f_z(x,y,z) = xye^{xz}$ so $h(z) = K$ and $f(x,y,z) = ye^{xz} + K$.