6.4 | DISCOVERY PROJECT: CAS PATTERNS IN INTEGRALS

This project can be completed anytime after you have studied Section 6.4 in the textbook.

In this project a computer algebra system is used to investigate indefinite integrals of families of functions. By observing the patterns that occur in the integrals of several members of the family, you will first guess, and then prove, a general formula for the integral of any member of the family.

1. (a) Use a computer algebra system to evaluate the following integrals.

(i) $\displaystyle\int \frac{1}{(x+2)(x+3)}\, dx$ (ii) $\displaystyle\int \frac{1}{(x+1)(x+5)}\, dx$

(iii) $\displaystyle\int \frac{1}{(x+2)(x-5)}\, dx$ (iv) $\displaystyle\int \frac{1}{(x+2)^2}\, dx$

(b) Based on the pattern of your responses in part (a), guess the value of the integral

$$\int \frac{1}{(x+a)(x+b)}\, dx$$

if $a \neq b$. What if $a = b$?

(c) Check your guess by asking your CAS to evaluate the integral in part (b). Then prove it using partial fractions or by differentiation.

2. (a) Use a computer algebra system to evaluate the following integrals.

(i) $\displaystyle\int \sin x \cos 2x\, dx$ (ii) $\displaystyle\int \sin 3x \cos 7x\, dx$ (iii) $\displaystyle\int \sin 8x \cos 3x\, dx$

(b) Based on the pattern of your responses in part (a), guess the value of the integral

$$\int \sin ax \cos bx\, dx$$

(c) Check your guess with a CAS. Then prove it by differentiation. For what values of a and b is it valid?

3. (a) Use a computer algebra system to evaluate the following integrals.

(i) $\displaystyle\int \ln x\, dx$ (ii) $\displaystyle\int x \ln x\, dx$ (iii) $\displaystyle\int x^2 \ln x\, dx$

(iv) $\displaystyle\int x^3 \ln x\, dx$ (v) $\displaystyle\int x^7 \ln x\, dx$

(b) Based on the pattern of your responses in part (a), guess the value of

$$\int x^n \ln x\, dx$$

(c) Use integration by parts to prove the conjecture that you made in part (b). For what values of n is it valid?

4. (a) Use a computer algebra system to evaluate the following integrals.

(i) $\displaystyle\int xe^x\, dx$ (ii) $\displaystyle\int x^2 e^x\, dx$ (iii) $\displaystyle\int x^3 e^x\, dx$

(iv) $\displaystyle\int x^4 e^x\, dx$ (v) $\displaystyle\int x^5 e^x\, dx$

(b) Based on the pattern of your responses in part (a), guess the value of $\int x^6 e^x\, dx$. Then use your CAS to check your guess.

(c) Based on the patterns in parts (a) and (b), make a conjecture as to the value of the integral

$$\int x^n e^x\, dx$$

when n is a positive integer.

(d) Use mathematical induction to prove the conjecture you made in part (c).