9.2 LABORATORY PROJECT: BÉZIER CURVES

This project can be completed anytime after you have studied Section 9.2 in the textbook.

Bézier curves are used in computer-aided design and are named after the French mathematician Pierre Bézier (1910–1999), who worked in the automotive industry. A cubic Bézier curve is determined by four *control points*, $P_0(x_0, y_0)$, $P_1(x_1, y_1)$, $P_2(x_2, y_2)$, and $P_3(x_3, y_3)$, and is defined by the parametric equations

$$x = x_0(1 - t)^3 + 3x_1t(1 - t)^2 + 3x_2t^2(1 - t) + x_3t^3$$

$$y = y_0(1 - t)^3 + 3y_1t(1 - t)^2 + 3y_2t^2(1 - t) + y_3t^3$$

where $0 \leqslant t \leqslant 1$. Notice that when $t = 0$ we have $(x, y) = (x_0, y_0)$ and when $t = 1$ we have $(x, y) = (x_3, y_3)$, so the curve starts at P_0 and ends at P_3.

1. Graph the Bézier curve with control points $P_0(4, 1)$, $P_1(28, 48)$, $P_2(50, 42)$, and $P_3(40, 5)$. Then, on the same screen, graph the line segments P_0P_1, P_1P_2, and P_2P_3. (Exercise 25 in Section 9.1 shows how to do this.) Notice that the middle control points P_1 and P_2 don't lie on the curve; the curve starts at P_0, heads toward P_1 and P_2 without reaching them, and ends at P_3.
2. From the graph in Problem 1 it appears that the tangent at P_0 passes through P_1 and the tangent at P_3 passes through P_2. Prove it.
3. Try to produce a Bézier curve with a loop by changing the second control point in Problem 1.
4. Some laser printers use Bézier curves to represent letters and other symbols. Experiment with control points until you find a Bézier curve that gives a reasonable representation of the letter C.
5. More complicated shapes can be represented by piecing together two or more Bézier curves. Suppose the first Bézier curve has control points P_0, P_1, P_2, P_3 and the second one has control points P_3, P_4, P_5, P_6. If we want these two pieces to join together smoothly, then the tangents at P_3 should match and so the points P_2, P_3, and P_4 all have to lie on this common tangent line. Using this principle, find control points for a pair of Bézier curves that represent the letter S.