

5.3

EVALUATING DEFINITE INTEGRALS

EXAMPLE A Evaluate $\int_3^6 \frac{dx}{x}$.

SOLUTION The given integral is an abbreviation for

$$\int_3^6 \frac{1}{x} dx$$

An antiderivative of $f(x) = 1/x$ is $F(x) = \ln |x|$ and, because $3 \leq x \leq 6$, we can write $F(x) = \ln x$. So

$$\begin{aligned} \int_3^6 \frac{1}{x} dx &= \ln x \Big|_3^6 = \ln 6 - \ln 3 \\ &= \ln \frac{6}{3} = \ln 2 \end{aligned}$$

EXAMPLE B Evaluate $\int \frac{\cos \theta}{\sin^2 \theta} d\theta$.

SOLUTION This indefinite integral isn't immediately apparent in Table 1, so we use trigonometric identities to rewrite the function before integrating:

$$\begin{aligned} \int \frac{\cos \theta}{\sin^2 \theta} d\theta &= \int \left(\frac{1}{\sin \theta} \right) \left(\frac{\cos \theta}{\sin \theta} \right) d\theta \\ &= \int \csc \theta \cot \theta d\theta = -\csc \theta + C \end{aligned}$$