

1.4 CALCULATING LIMITS

EXAMPLE A If

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8 - 2x & \text{if } x < 4 \end{cases}$$

determine whether $\lim_{x \rightarrow 4} f(x)$ exists.

SOLUTION Since $f(x) = \sqrt{x-4}$ for $x > 4$, we have

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} \sqrt{x-4} = \sqrt{4-4} = 0$$

Since $f(x) = 8 - 2x$ for $x < 4$, we have

$$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} (8 - 2x) = 8 - 2 \cdot 4 = 0$$

The right- and left-hand limits are equal. Thus, the limit exists and

$$\lim_{x \rightarrow 4} f(x) = 0$$

The graph of f is shown in Figure 1. ■

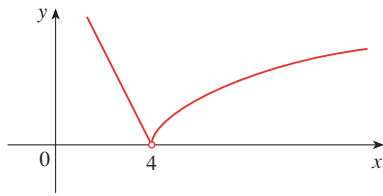


FIGURE 1

■ It is shown in Example 10 in Section 1.3 that $\lim_{x \rightarrow 0^+} \sqrt{x} = 0$.