

7.5 APPLICATIONS TO PHYSICS AND ENGINEERING

EXAMPLE A Find the centroid of the region bounded by the curves $y = \cos x$, $y = 0$, $x = 0$, and $x = \pi/2$.

SOLUTION The area of the region is

$$A = \int_0^{\pi/2} \cos x \, dx = \sin x \Big|_0^{\pi/2} = 1$$

so Formulas 12 give

$$\begin{aligned} \bar{x} &= \frac{1}{A} \int_0^{\pi/2} x f(x) \, dx = \int_0^{\pi/2} x \cos x \, dx \\ &= x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x \, dx \quad (\text{by integration by parts}) \\ &= \frac{\pi}{2} - 1 \end{aligned}$$

$$\begin{aligned} \bar{y} &= \frac{1}{A} \int_0^{\pi/2} \frac{1}{2} [f(x)]^2 \, dx = \frac{1}{2} \int_0^{\pi/2} \cos^2 x \, dx \\ &= \frac{1}{4} \int_0^{\pi/2} (1 + \cos 2x) \, dx = \frac{1}{4} \left[x + \frac{1}{2} \sin 2x \right]_0^{\pi/2} \\ &= \frac{\pi}{8} \end{aligned}$$

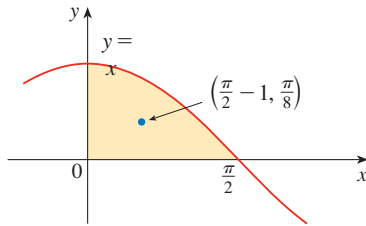


FIGURE 1

The centroid is $(\frac{\pi}{2} - 1, \frac{\pi}{8})$ and is shown in Figure 1. ■