Case Studies in Mathematical Modeling

A **mathematical model** is a mathematical description (often by means of a function or an equation) of a real-world phenomenon, such as the size of a population, the speed of a falling object, the frequency of a particular gene, the concentration of an antibiotic in a patient, or the life expectancy of a person at birth. The purpose of the model is to understand the phenomenon and perhaps to make predictions about future behavior.

Figure 1 illustrates the process of mathematical modeling. Given a real-world problem, the first task is to formulate a mathematical model by identifying and naming the relevant quantities and making assumptions that simplify the phenomenon enough to make it mathematically tractable. We use our knowledge of the biological situation and our mathematical skills to obtain equations that relate the quantities. In situations where there is no physical law to guide us, we may need to collect data (either from a library or the Internet or by conducting our own experiments) and examine the data to discern patterns.

The second stage is to apply the mathematics that we know (such as the calculus that will be developed throughout this book) to the mathematical model that we have formulated in order to derive mathematical conclusions. Then, in the third stage, we take those mathematical conclusions and interpret them as information about the original biological phenomenon by way of offering explanations or making predictions. The final step is to test our predictions by checking them against new real data. If the predictions don't compare well with reality, we need to refine our model or to formulate a new model and start the cycle again.

A mathematical model is never a completely accurate representation of a physical situation—it is an idealization. Picasso once said that "art is a lie that makes us realize truth." The same could be said about mathematical models. A good model simplifies reality enough to permit mathematical calculations, but is nevertheless realistic enough to teach us something important about the real world. Because models are simplifications, however, it is always important to keep their limitations in mind. In the end, Mother Nature has the final say.

Throughout this book we will explore a variety of different mathematical models from the life sciences. In each case we provide a brief description of the real-world problem as well as a brief mention of the real-world predictions that result from the mathematical analysis. Nevertheless, the main body of this text is designed to teach important mathematical concepts and techniques and therefore its focus is primarily on the center portion of Figure 1.

To better illustrate the entirety of the modeling process, however, we also provide a pair of *case studies in mathematical modeling.* Each case study is an extended, self-contained example of mathematical modeling from the scientific literature. In the following pages the real-world problem at the center of each case study is introduced as motivation for learning the mathematics in this book. Then, throughout subsequent chapters, these case studies are periodically revisited as we develop our mathematical skills further. In doing so, we illustrate how these mathematical skills help to address real-world problems. Additional case studies can be found on the website www.stewartcalculus.com.

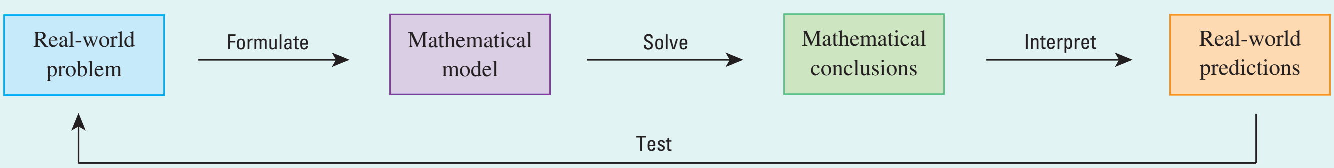

FIGURE 1 The modeling process